Chapter 9 Rational Functions

Section 9.1 Exploring Rational Functions Using Transformations

Section 9.1 Page 442 Question 1

Compare each graph to the function form $y = \frac{a}{x-h} + k$.

a) Since the graph of the rational function has a vertical asymptote at $x = –1$ and a horizontal asymptote at $y = 0$, $h = –1$ and $k = 0$. Then, the function is of the form $y = \frac{a}{x+1}$, which is $B(x) = \frac{2}{x+1}$.

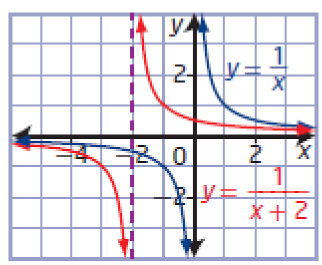
b) Since the graph of the rational function has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = –1$, $h = 0$ and $k = –1$. Then, the function is of the form $y = \frac{a}{x} - 1$, which is $A(x) = \frac{2}{x} - 1$.

c) Since the graph of the rational function has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 1$, $h = 0$ and $k = 1$. Then, the function is of the form $y = \frac{a}{x} + 1$, which is $D(x) = \frac{2}{x} + 1$.

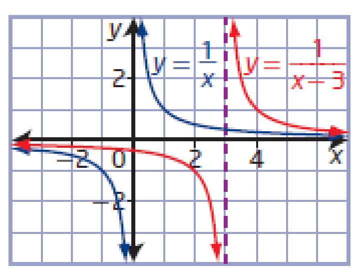
d) Since the graph of the rational function has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 0$, $h = 1$ and $k = 0$. Then, the function is of the form $y = \frac{a}{x-1}$, which is $C(x) = \frac{2}{x-1}$.

Section 9.1 Page 442 Question 2

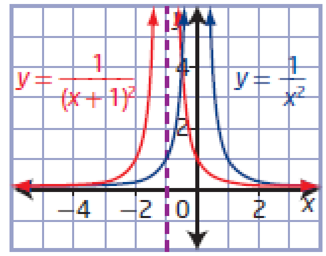
a) The base function for $y = \frac{1}{x+2}$ is $y = \frac{1}{x}$. Compare the function to the form $y = \frac{a}{x-h} + k$: $a = 1$, $h = –2$, and $k = 0$.
The graph of the base function must be translated 2 units to the left. So, the vertical asymptote is at $x = –2$ and the horizontal asymptote is still at $y = 0$.



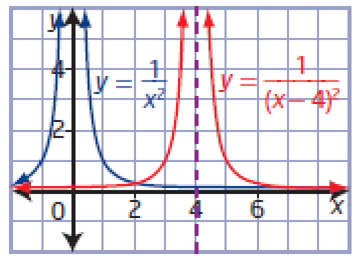
b) The base function for $y = \frac{1}{x-3}$ is $y = \frac{1}{x}$. Compare the function to the form $y = \frac{a}{x-h} + k$: $a = 1$, $h = 3$, and $k = 0$.
The graph of the base function must be translated 3 units to the right. So, the vertical asymptote is at $x = 3$ and the horizontal asymptote is still at $y = 0$.



c) The base function for $y = \dfrac{1}{(x+1)^2}$ is $y = \dfrac{1}{x^2}$. Compare the function to the form $y = \dfrac{a}{(x-h)^2} + k$: $a = 1$, $h = –1$, and $k = 0$. The graph of the base function must be translated 1 unit to the left. So, the vertical asymptote is at $x = –1$ and the horizontal asymptote is still at $y = 0$.

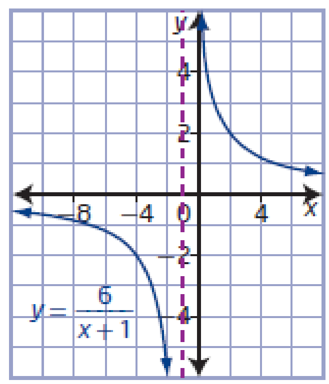


d) The base function for $y = \dfrac{1}{(x-4)^2}$ is $y = \dfrac{1}{x^2}$. Compare the function to the form $y = \dfrac{a}{(x-h)^2} + k$: $a = 1$, $h = 4$, and $k = 0$. The graph of the base function must be translated 4 units to the right. So, the vertical asymptote is at $x = 4$ and the horizontal asymptote is still at $y = 0$.

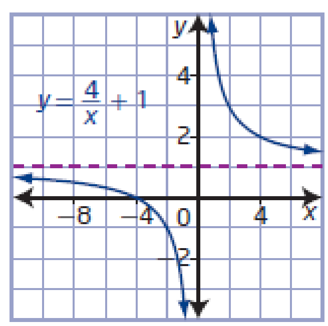


Section 9.1 Page 442 Question 3

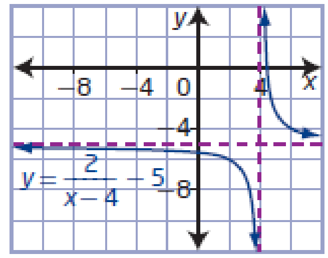
a) For $y = \dfrac{6}{x+1}$, $a = 6$, $h = –1$, and $k = 0$. The graph of the base function $y = \dfrac{1}{x}$ must be stretched vertically by a factor of 6 and translated 1 unit to the left. The domain is $\{x \mid x \neq -1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$. There is no x-intercept. The y-intercept is 6. The vertical asymptote is at $x = –1$ and the horizontal asymptote is at $y = 0$.



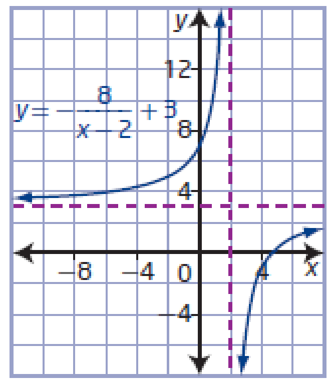
b) For $y = \dfrac{4}{x} + 1$, $a = 4$, $h = 0$, and $k = 1$. The graph of the base function $y = \dfrac{1}{x}$ must be stretched vertically by a factor of 4 and translated 1 unit up. The domain is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 1, y \in \mathrm{R}\}$. The x-intercept is –4. There is no y-intercept. The vertical asymptote is at $x = 0$ and the horizontal asymptote is at $y = 1$.



c) For $y = \frac{2}{x-4} - 5$, $a = 2$, $h = 4$, and $k = –5$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 2 and translated 4 units to the right and 5 units down. The domain is $\{x \mid x \neq 4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -5, y \in \mathrm{R}\}$. The x-intercept is $\frac{22}{5}$, or 4.4. The y-intercept is $-\frac{11}{2}$, or –5.5. The vertical asymptote is at $x = 4$ and the horizontal asymptote is at $y = –5$.

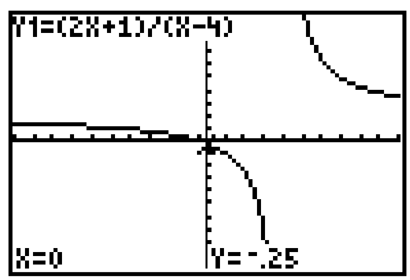


d) For $y = -\frac{8}{x-2} + 3$, $a = –8$, $h = 2$, and $k = 3$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 8, reflected in the x-axis, and translated 2 units to the right and 3 units up. The domain is $\{x \mid x \neq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 3, y \in \mathrm{R}\}$. The x-intercept is $\frac{14}{3}$. The y-intercept is 7. The vertical asymptote is at $x = 2$ and the horizontal asymptote is at $y = 3$.

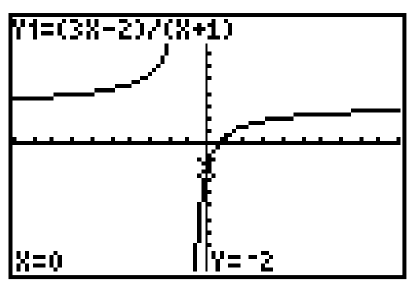


Section 9.1 Page 442 Question 4

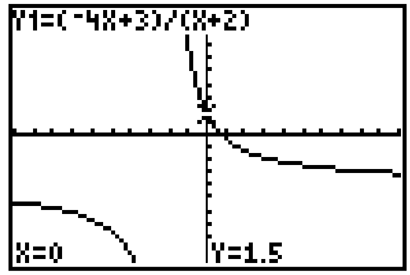
a) For $y = \frac{2x+1}{x-4}$, the vertical asymptote is at $x = 4$, the horizontal asymptote is at $y = 2$, the x-intercept is –0.5, and the y-intercept is –0.25.



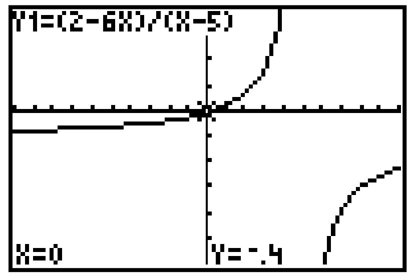
b) For $y = \frac{3x-2}{x+1}$, the vertical asymptote is at $x = –1$, the horizontal asymptote is at $y = 3$, the x-intercept is about 0.67, and the y-intercept is –2.



c) For $y = \dfrac{-4x+3}{x+2}$, the vertical asymptote is at $x = -2$, the horizontal asymptote is at $y = -4$, the x-intercept is 0.75, and the y-intercept is 1.5.



d) For $y = \dfrac{2-6x}{x-5}$, the vertical asymptote is at $x = 5$, the horizontal asymptote is at $y = -6$, the x-intercept is about 0.33, and the y-intercept is –0.4.

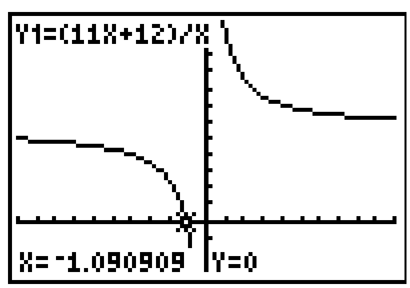


Section 9.1 Page 442 Question 5

a)
$$
\begin{aligned}
y &= \frac{11x+12}{x} \\
&= 11 + \frac{12}{x} \\
&= \frac{12}{x} + 11
\end{aligned}
$$

For $y = \dfrac{12}{x} + 11$, $a = 12$, $h = 0$, and $k = 11$. The vertical asymptote is at $x = 0$, and the horizontal asymptote is at $y = 11$.
Substitute $y = 0$.

$$
\begin{aligned}
y &= \frac{12}{x} + 11 \\
0 &= \frac{12}{x} + 11 \\
-11 &= \frac{12}{x} \\
x &= -\frac{12}{11}
\end{aligned}
$$



The x-intercept is $-\dfrac{12}{11}$, or about –1.09, and there is no y-intercept.

b) $y = \dfrac{x}{x+8}$

$= \dfrac{x+8-8}{x+8}$

$= \dfrac{x+8}{x+8} - \dfrac{8}{x+8}$

$= 1 - \dfrac{8}{x+8}$

$= -\dfrac{8}{x+8} + 1$

For $y = -\dfrac{8}{x+8} + 1$, $a = –8$, $h = –8$, and $k = 1$. The vertical asymptote is at $x = –8$, and the horizontal asymptote is at $y = 1$.

Substitute $y = 0$.

$y = -\dfrac{8}{x+8} + 1$

$0 = -\dfrac{8}{x+8} + 1$

$\dfrac{8}{x+8} = 1$

$x = 0$

Substitute $x = 0$.

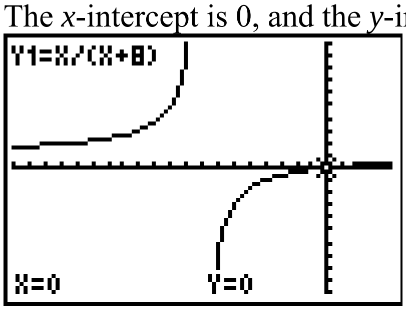
$y = -\dfrac{8}{x+8} + 1$

$= -\dfrac{8}{0+8} + 1$

$= –1 + 1$

$= 0$

The x-intercept is 0, and the y-intercept is 0.



c) $y = \dfrac{-x-2}{x+6}$

$= \dfrac{-x-6+6-2}{x+6}$

$= \dfrac{-(x+6)+4}{x+6}$

$= \dfrac{-(x+6)}{x+6} + \dfrac{4}{x+6}$

$= -1 + \dfrac{4}{x+6}$

For $y = \frac{4}{x+6} - 1$, $a = 4$, $h = –6$, and $k = –1$. The vertical asymptote is at $x = –6$, and the horizontal asymptote is at $y = –1$.

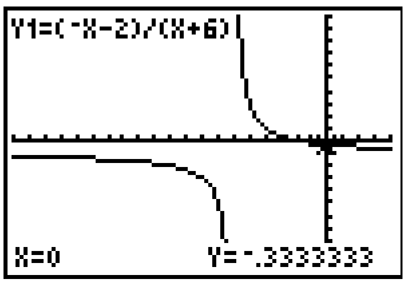
Substitute $y = 0$.

$$y = \frac{4}{x+6} - 1$$
$$0 = \frac{4}{x+6} - 1$$
$$1 = \frac{4}{x+6}$$
$$x + 6 = 4$$
$$x = –2$$

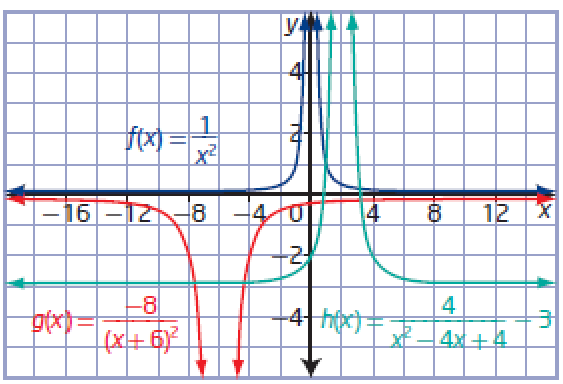
Substitute $x = 0$.

$$y = \frac{4}{x+6} - 1$$
$$= \frac{4}{0+6} - 1$$
$$= \frac{2}{3} - 1$$
$$= -\frac{1}{3}$$

The x-intercept is –2, and the y-intercept is $-\frac{1}{3}$, or about –0.33.



Section 9.1 Page 442 Question 6

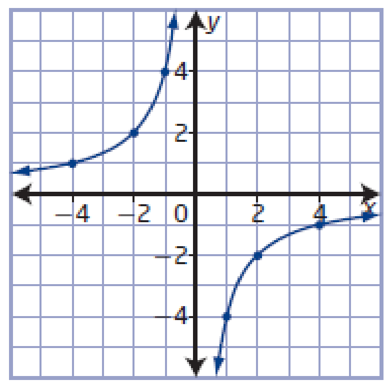


Characteristic	$f(x) = \frac{1}{x^2}$	$g(x) = \frac{-8}{(x+6)^2}$	$h(x) = \frac{4}{x^2 - 4x + 4} - 3$
Non-permissible value	$x = 0$	$x = –6$	$x = −2$
Behaviour near non-permissible value	As x approaches 0, $\lvert y\rvert$ becomes very large.	As x approaches –6, $\lvert y\rvert$ becomes very large.	As x approaches −2, $\lvert y\rvert$ becomes very large.
End behaviour	As $\lvert x\rvert$ becomes very large, y approaches 0.	As $\lvert x\rvert$ becomes very large, y approaches 0.	As $\lvert x\rvert$ becomes very large, y approaches –3.

Domain	$\{x \mid x \neq 0, x \in \mathrm{R}\}$	$\{x \mid x \neq -6, x \in \mathrm{R}\}$	$\{x \mid x \neq -2, x \in \mathrm{R}\}$
Range	$\{y \mid y > 0, y \in \mathrm{R}\}$	$\{y \mid y < 0, y \in \mathrm{R}\}$	$\{y \mid y > -3, y \in \mathrm{R}\}$
Equation of vertical asymptote	$x = 0$	$x = -6$	$x = -2$
Equation of horizontal asymptote	$y = 0$	$y = 0$	$y = -3$

Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consists of all real numbers except for a single value. The range of each function consists of a restricted set of the real numbers. $|y|$ becomes very large for each function when the values of x approach the non-permissible value for the function.

Section 9.1　Page 443　Question 7



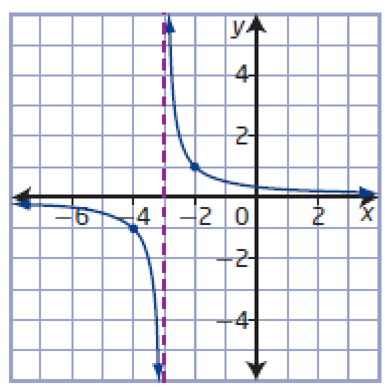
a) From the graph, the vertical asymptote is at $x = 0$ and the horizontal asymptote is at $y = 0$. So, $h = 0$ and $k = 0$.

Then, the equation of the function is of the form $y = \frac{a}{x}$.

Use one of the given points, say (2, –2), to determine the value of a.

$$-2 = \frac{a}{2}$$
$$a = -4$$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = -\frac{4}{x}$.



b) From the graph, the vertical asymptote is at $x = -3$ and the horizontal asymptote is at $y = 0$. So, $h = -3$ and $k = 0$. Then, the equation of the function is of the form

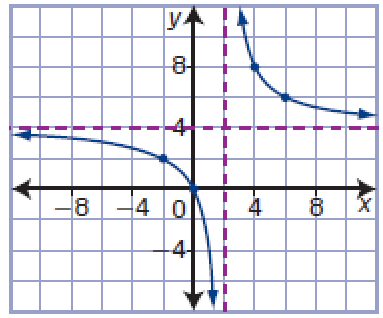
$$y = \frac{a}{x+3}.$$

Use one of the given points, say (–2, 1), to determine the value of a.

$$1 = \frac{a}{-2+3}$$
$$a = 1$$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = \frac{1}{x+3}$.

c) From the graph, the vertical asymptote is at $x = 2$ and the horizontal asymptote is at $y = 4$. So, $h = 2$ and $k = 4$. Then, the equation of the function is of the form



$$y = \frac{a}{x-2} + 4.$$

Use one of the given points, say (4, 8), to determine the value of a.

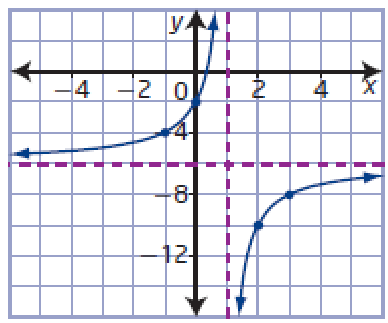
$$8 = \frac{a}{4-2} + 4$$

$$4 = \frac{a}{2}$$

$$a = 8$$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = \frac{8}{x-2} + 4$.

d) From the graph, the vertical asymptote is at $x = 1$ and the horizontal asymptote is at $y = –6$. So, $h = 1$ and $k = –6$. Then, the equation of the function is of the form



$$y = \frac{a}{x-1} - 6.$$

Use one of the given points, say (0, –2), to determine the value of a.

$$-2 = \frac{a}{0-1} - 6$$

$$4 = -a$$

$$a = -4$$

The equation of the function in the form $y = \frac{a}{x-h} + k$ is $y = -\frac{4}{x-1} - 6$.

Section 9.1 Page 443 Question 8

a) Given: $y = \frac{a}{x-7} + k$ passing through points (10, 1) and (2, 9)

For (10, 1),

$$y = \frac{a}{x-7} + k$$

$$1 = \frac{a}{10-7} + k$$

$$1 = \frac{a}{3} + k$$

$3 = a + 3k$ ①

For (2, 9),

$$y = \frac{a}{x-7} + k$$

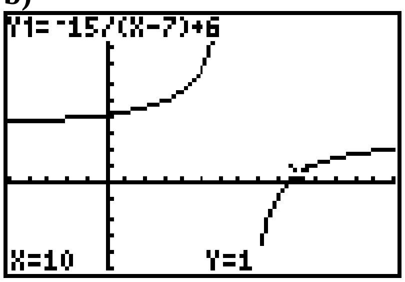
$$9 = \frac{a}{2-7} + k$$

$$9 = \frac{a}{-5} + k$$

$–45 = a – 5k$ ②

Solve the system of equations.

$3 = a + 3k$
$\underline{-45 = a - 5k}$
$48 = 8k \qquad ① - ②$
$k = 6$
Substitute $k = 6$ into ①.
$3 = a + 3k$
$3 = a + 3(6)$
$a = -15$
The equation of the function is $y = -\dfrac{15}{x-7} + 6$.

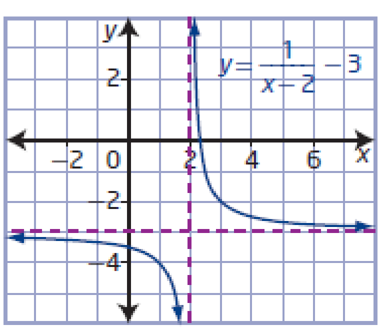
b)


Section 9.1 Page 443 Question 9

Examples:
a) For asymptotes at $x = 2$ and $y = -3$, $h = 2$ and $k = -3$. Choose $a = 1$, then the equation of the function in the form $y = \dfrac{a}{x-h} + k$ is $y = \dfrac{1}{x-2} - 3$.

Then, rewrite in the form $y = \dfrac{p(x)}{q(x)}$:

$$\begin{aligned} y &= \frac{1}{x-2} - 3 \\ &= \frac{1}{x-2} - \frac{3(x-2)}{x-2} \\ &= \frac{1-3x+2}{x-2} \\ &= \frac{3-3x}{x-2} \end{aligned}$$

b)


The domain is $\{x \mid x \neq 2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -3, y \in \mathrm{R}\}$.

c) There are many possible functions that meet the given criteria, since any value of *a* (other than 0) will result in the same equations for the asymptotes.

Section 9.1 Page 443 Question 10

a) In the fourth line, Mira incorrectly factored –3 from –3*x* – 21. She should have grouped –3*x* + 21. The corrected solution is

$$y=\frac{2-3x}{x-7}$$
$$y=\frac{-3x+2}{x-7}$$
$$y=\frac{-3x+21-21+2}{x-7}$$
$$y=\frac{-3(x-7)-19}{x-7}$$
$$y=\frac{-3(x-7)}{x-7}-\frac{19}{x-7}$$
$$y=-3-\frac{19}{x-7}$$
$$y=-\frac{19}{x-7}-3$$

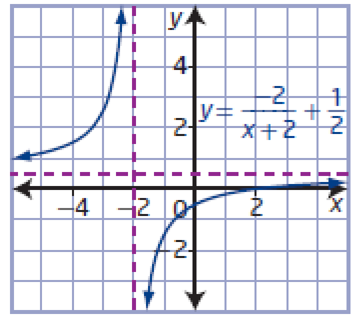
b) Example: Without technology, Mira could have discovered her error by substituting the same value of *x* into each form of the function. With technology, Mira could have graphed the two functions to see if they were the same.

Section 9.1 Page 443 Question 11

a)

$$y=\frac{x-2}{2x+4}$$
$$y=\frac{x-2}{2(x+2)}$$
$$y=\frac{x+2-4}{2(x+2)}$$
$$y=\frac{x+2}{2(x+2)}-\frac{4}{2(x+2)}$$
$$y=\frac{1}{2}-\frac{2}{x+2}$$
$$y=-\frac{2}{x+2}+\frac{1}{2}$$

b) For $y = -\frac{2}{x+2} + \frac{1}{2}$, $a = -2$, $h = -2$, and $k = \frac{1}{2}$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 2, reflected in the x-axis, and translated 2 units to the left and $\frac{1}{2}$ unit up.



Section 9.1 Page 443 Question 12

Determine the intercepts.

Substitute $y = 0$.

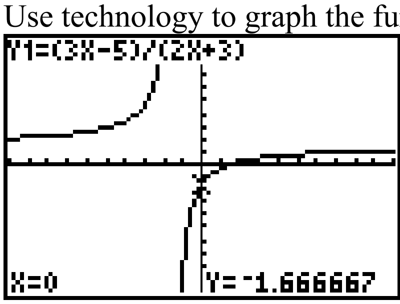
$$\begin{aligned} y &= \frac{3x-5}{2x+3} \\ {\color{red}0} &= \frac{3x-5}{2x+3} \\ 0 &= 3x - 5 \\ x &= \frac{5}{3} \end{aligned}$$

Substitute $x = 0$.

$$\begin{aligned} y &= \frac{3x-5}{2x+3} \\ &= \frac{3({\color{red}0})-5}{2({\color{red}0})+3} \\ &= -\frac{5}{3} \end{aligned}$$

The x-intercept is $\frac{5}{3}$, and the y-intercept is $-\frac{5}{3}$.

Use technology to graph the function.



The asymptotes are located at $x = -\frac{3}{2}$ and $y = \frac{3}{2}$.

Section 9.1 Page 443 Question 13

For the function $N(p) = \frac{500\,000}{p}$, as the value of p increases, the value of N decreases.
This means that as the average price of a home increases, the number of buyers looking to buy a home decreases.

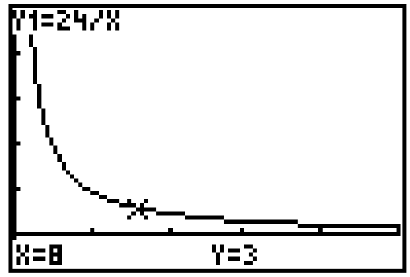
Section 9.1 Page 444 Question 14

a) For a rectangle with constant area of 24 cm^2,

$A = lw$

$24 = lw$

$l = \dfrac{24}{w}$

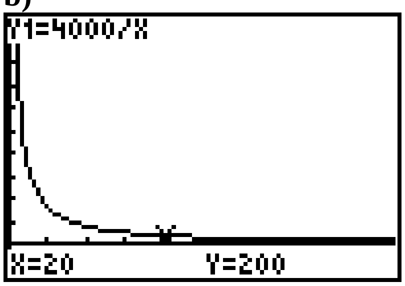


b) As the width increases, the length decreases to maintain an area of 24 cm^2.

Section 9.1 Page 444 Question 15

a) Let x represent the number of students who contribute. Let y represent the average amount required per student to meet the goal. Then, a function to model this situation is $xy = 4000$, or $y = \dfrac{4000}{x}$.

b)



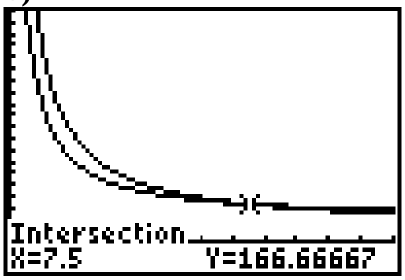
c) As the number of students that contribute increases, the average amount required by student decreases.

d) If the student council also received a \$1000 donation from a local business, the function becomes $y = \dfrac{4000}{x} + 1000$. This represents a vertical translation of 1000 units up of the original function graph.

Section 9.1 Page 444 Question 16

a) Let C represent the average cost per year. Let t represent the time, in years. Then, a function to model freezer one is $C = \dfrac{500 + 100t}{t}$ and a function to model freezer two is $C = \dfrac{800 + 60t}{t}$.

b)



c) Both graphs have a vertical asymptote at $x = 0$, but different horizontal asymptotes. The average cost for freezer one approaches \$100/year, while the average cost for freezer two approaches \$60/year. The graph shows that the more years you run the freezer, the less the average cost per year is. Freezer one is cheaper to run for a short amount of time, while freezer two is cheaper if you run it for a longer period of time.

d) The point of intersection of the two graphs can help Hanna decide which model to choose. If Hanna wants to run the freezer for more than 7.5 years, she should choose the second model. Otherwise, she is better off with the first one.

Section 9.1 Page 444 Question 17

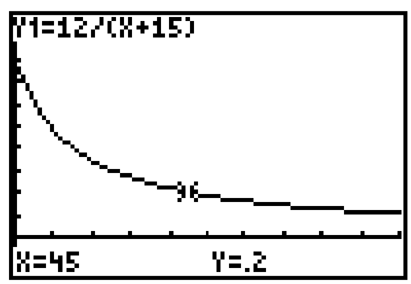
a) An equation for the current in the given circuit is $I = \dfrac{12}{15+x}$.

b) Since the variable resistor can be set anywhere from 0 Ω to 100 Ω, an appropriate domain is $\{x \mid 0 \le x \le 100, x \in \mathrm{R}\}$. The graph does not have a vertical asymptote for this domain.

c) Substitute $I = 0.2$.

$$I = \frac{12}{15+x}$$
$$0.2 = \frac{12}{15+x}$$
$$0.2(15 + x) = 12$$
$$15 + x = 60$$
$$x = 45$$

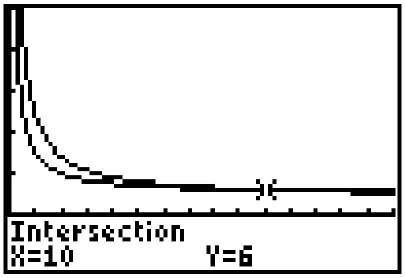
A setting of 45 Ω is required.



d) An equation for the current in the circuit without the bulb is $I = \dfrac{12}{x}$. The vertical asymptote is at $x = 0$, so the domain must change to $\{x \mid 0 < x \le 100, x \in \mathrm{R}\}$.

Section 9.1 Page 445 Question 18

a) Let C represent the average cost per hour. Let t represent the rental time, in hours. Then, a function to model renting from store one is $C = \dfrac{20+4t}{t}$ and a function to model renting from store two is $C = \dfrac{10+5t}{t}$.



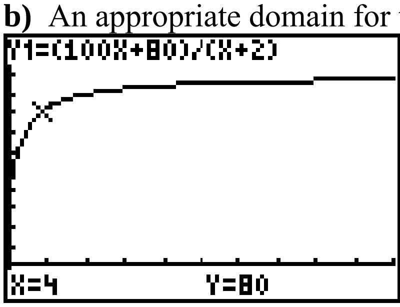
b) Both graphs have a vertical asymptote at $x = 0$, but different horizontal asymptotes. The average cost for renting from store one approaches \$4/h, while the average cost for renting from store two approaches \$5/h. The graph shows that the longer you rent the bike, the less the average cost per hour is. Store two is cheaper to rent for a short amount of time, while store one is cheaper if you rent for a longer period of time.

c) No. The point of intersection of the two graphs indicates that if you rent a bike for less than 10 h, then you should choose store two. Otherwise, choose store one.

Section 9.1 Page 445 Question 19

a) Let v represent the average speed, in kilometres per hour, over the entire trip and t represent the time, in hours, since leaving the construction zone. Then, an equation for v as a function of t for this situation is $v = \dfrac{80+100t}{2+t}$.

b) An appropriate domain for this situation is $\{t \mid t > 0, t \in \mathrm{R}\}$.



c) The equation of the vertical asymptote is $t = -2$ and the equation of the horizontal asymptote is $v = 100$. The vertical asymptote does not mean anything in this context, since time cannot be negative. The horizontal asymptote means that the average speed gets closer and closer to 100 km/h but never reaches it.

d) Substitute $v = 80$.

$$v = \frac{80+100t}{2+t}$$
$$80 = \frac{80+100t}{2+t}$$
$$80(2 + t) = 80 + 100t$$
$$160 + 80t = 80 + 100t$$
$$-20t = -80$$
$$t = 4$$

The truck will have to drive 4 h after leaving the construction zone before its average speed is 80 km/h.

e) Example: By including an application on a GPS unit that calculates the average speed over the entire trip, a driver could adjust his/her speed accordingly to maintain a target speed that provides the best fuel economy for his/her vehicle.

Section 9.1 Page 445 Question 20

Given: vertical asymptote at $x = 6$, horizontal asymptote at $y = -4$, and x-intercept of -1

For a function of the form $y = \frac{a}{x-h}+k$, $h = 6$ and $k = -4$. Then, the equation becomes $y = \frac{a}{x-6}-4$. Use the point $(-1, 0)$ to determine the value of a.

$$0 = \frac{a}{-1-6}-4$$
$$4 = -\frac{a}{7}$$
$$a = -28$$

Rewrite the function $y = -\frac{28}{x-6}-4$ in the form $y = \frac{ax+b}{cx+d}$.

$$\begin{aligned} y &= -\frac{28}{x-6}-4 \\ &= -\frac{28}{x-6}-\frac{4(x-6)}{x-6} \\ &= \frac{-28-4x+24}{x-6} \\ &= \frac{-4x-4}{x-6} \end{aligned}$$

Section 9.1 Page 445 Question 21

a)

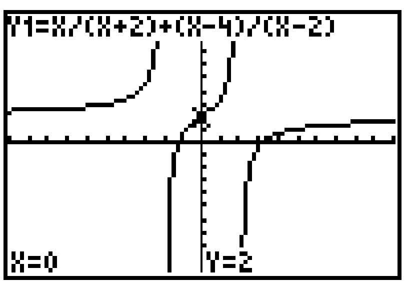
$$\begin{aligned}
f(x) &= \frac{x-3}{x+1} \\
y &= \frac{x-3}{x+1} \\
x &= \frac{y-3}{y+1} \\
x(y+1) &= y-3 \\
xy+x &= y-3 \\
xy-y &= -x-3 \\
y(x-1) &= -x-3 \\
y &= \frac{-x-3}{x-1} \\
f^{-1}(x) &= \frac{-x-3}{x-1}
\end{aligned}$$

b)

$$\begin{aligned}
f(x) &= \frac{2x}{x-5}+4 \\
y &= \frac{2x}{x-5}+4 \\
x &= \frac{2y}{y-5}+4 \\
(x-4)(y-5) &= 2y \\
xy-4y-5x+20 &= 2y \\
xy-6y &= 5x-20 \\
y(x-6) &= 5x-20 \\
y &= \frac{5x-20}{x-6} \\
f^{-1}(x) &= \frac{5x-20}{x-6}
\end{aligned}$$

Section 9.1 Page 445 Question 22

Use graphing technology to graph $y = \frac{x}{x+2} + \frac{x+4}{x-2}$.



The graph has three branches separated by vertical asymptotes at $x = –2$ and $x = 2$. It also appears to have a horizontal asymptote at $y = 2$ for $x < –2$ and $x > 2$.

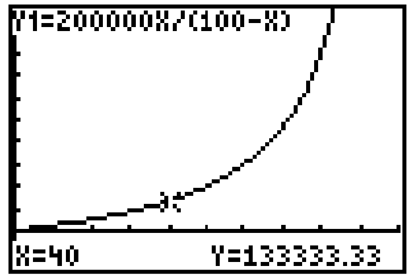
Section 9.1 Page 445 Question C1

Example: If the equation of the rational function is given in the form $y = \frac{a}{x-h} + k$, then using transformations is no different than with any other function. However, if the equation of the function is not in this form, it is more difficult to manipulate the equation to determine the transformations that have been applied.

Section 9.1 Page 445 Question C2

a) For the function $C(p) = \frac{200\,000p}{100 - p}$, an appropriate domain is $\{p \mid 0 \le p < 100, p \in \mathrm{R}\}$. The function is not defined at $p = 100$, meaning that 100% of the emissions can never be eliminated.

b) The shape of the graph indicates that as the percent of emissions eliminated increases, so does the cost.



c) For $p = 80$,

$$C(p) = \frac{200\,000p}{100 - p}$$

$$C(80) = \frac{200\,000(80)}{100 - 80}$$

$$C(80) = 800\,000$$

For $p = 40$,

$$C(p) = \frac{200\,000p}{100 - p}$$

$$C(40) = \frac{200\,000(40)}{100 - 40}$$

$$C(40) = 133\,333.33$$

It costs 6 times as much to eliminate 80% as it does to eliminate 40%.

d) Is it not possible to completely eliminate all of the emissions according to this model because the graph of the function has a vertical asymptote at $p = 100$.

Section 9.1 Page 445 Question C3

For $y = \frac{2}{x-3} + 4$, $a = 2$, $h = 3$, and $k = 4$. For $y = 2\sqrt{x-3} + 4$, $a = 2$, $h = 3$, and $k = 4$.
Example: Both functions are vertically stretched by a factor of 2 and translated 3 units right and 4 units up. In the case of the rational function, the values of the parameters h and k represent the locations of asymptotes. For the square root function, the point (h, k) gives the location of the endpoint of the graph.

Section 9.2 Analysing Rational Functions

Section 9.2 Page 451 Question 1

a)

Characteristic	$y = \dfrac{x - 4}{x^2 - 6x + 8}$
Non-permissible value(s)	$x = 2$, $x = 4$
Feature exhibited at each non-permissible value	vertical asymptote, point of discontinuity
Behaviour near each non-permissible value	As x approaches 2, $\|y\|$ becomes very large. As x approaches 4, y approaches 0.5.
Domain	$\{x \mid x \neq 2, 4, x \in \mathrm{R}\}$
Range	$\{y \mid y \neq 0, 0.5, y \in \mathrm{R}\}$

b) In factored form, $y = \dfrac{x-4}{x^2-6x+8}$ is $y = \dfrac{x-4}{(x-4)(x-2)}$. There is a vertical asymptote at $x = 2$ because $x - 2$ is factor of the denominator only. There is a point of discontinuity at (4, 0.5) because $x - 4$ is a factor of both the numerator and the denominator.

Section 9.2 Page 451 Question 2

Examples:

a) In factored form, $y = \dfrac{x^2-3x}{x}$ is $y = \dfrac{x(x-3)}{x}$. The non-permissible value is $x = 0$.

x	y
−1.5	−4.5
−1.0	−4.0
−0.5	−3.5
0.5	−2.5
1.0	−2.0
1.5	−1.5

Since the function does not increase or decrease drastically as x approaches the non-permissible value, it must be a point of discontinuity.

b) In factored form, $y = \dfrac{x^2-3x-10}{x-2}$ is $y = \dfrac{(x-5)(x+2)}{x-2}$. The non-permissible value is $x = 2$.

x	y
1.7	40.7
1.8	60.8
1.9	120.9
2.1	−118.9
2.2	−58.8
2.3	−38.7

Since the function changes sign at the non-permissible value and $|y|$ increases, it must be a vertical asymptote.

c) In factored form, $y = \dfrac{3x^2 + 4x - 4}{x + 4}$ is $y = \dfrac{(3x-2)(x+2)}{x+4}$. The non-permissible value is $x = -4$.

x	y
–3.7	74.23
–3.8	120.6
–3.9	260.3
–4.1	–300.3
–4.2	–160.6
–4.3	–114.23

Since the function changes sign at the non-permissible value and $|y|$ increases, it must be a vertical asymptote.

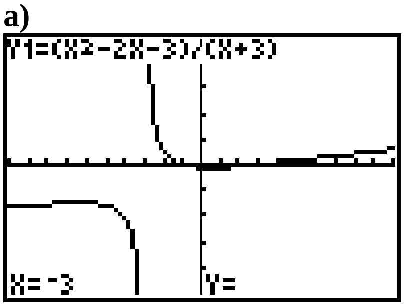
d) In factored form, $y = \dfrac{5x^2 + 4x - 1}{5x - 1}$ is $y = \dfrac{(5x-1)(x+1)}{5x-1}$. The non-permissible value is $x = 0.2$.

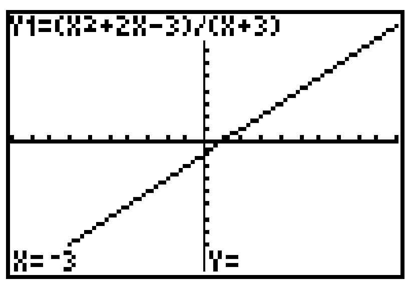
x	y
0.17	1.17
0.18	1.18
0.19	1.19
0.21	1.21
0.22	1.22
0.23	1.23

Since the function does not increase or decrease drastically as x approaches the non-permissible value, it must be a point of discontinuity.

Section 9.2 Page 451 Question 3

a)





Both of the functions have a non-permissible value of –3. However, the graph of $f(x)$ has a vertical asymptote, while the graph of $g(x)$ has a point of discontinuity.

b) In factored form, $f(x) = \dfrac{x^2 - 2x - 3}{x + 3}$ is $f(x) = \dfrac{(x-3)(x+1)}{x+3}$. In factored form $g(x) = \dfrac{x^2 + 2x - 3}{x + 3}$ is $g(x) = \dfrac{(x+3)(x-1)}{x+3}$.

The graph of $f(x)$ has a vertical asymptote at $x = -3$, since $x + 3$ is a factor of only the denominator.

Use the simplified form, $g(x) = x - 1$, $x \neq -3$, to determine the coordinates of the point of discontinuity. Substituting $x = -3$, the graph of $g(x)$ has a point of discontinuity at $(-3, -4)$. This is because $x + 3$ is a factor of both the numerator and the denominator.

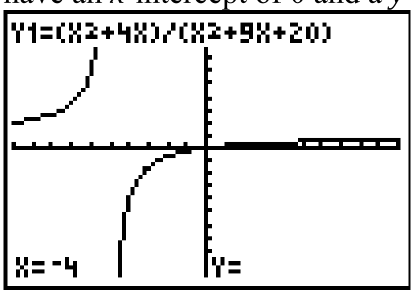
Section 9.2 Page 452 Question 4

a) $y = \dfrac{x^2 + 4x}{x^2 + 9x + 20}$

$= \dfrac{x(x+4)}{(x+4)(x+5)}$

$= \dfrac{x}{x+5}, x \neq -4, 5$

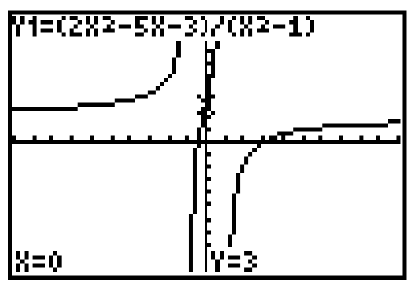
The graph will have a vertical asymptote at $x = -5$. Substituting $x = -4$, the graph will have a point of discontinuity at (–4, –4). By substituting $y = 0$ and $x = 0$, the graph will have an x-intercept of 0 and a y-intercept of 0, respectively.



b) $y = \dfrac{2x^2 - 5x - 3}{x^2 - 1}$

$= \dfrac{(2x+1)(x-3)}{(x+1)(x-1)}, x \neq -1, 1$

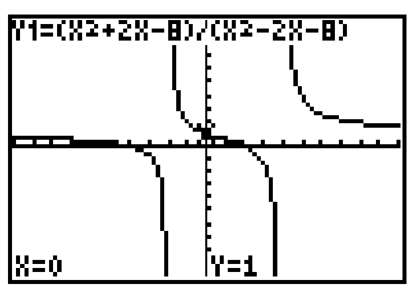
The graph will have vertical asymptotes at $x = -1$ and $x = 1$. The graph will have no points of discontinuity. By substituting $y = 0$ and $x = 0$, the graph will have x-intercepts of –0.5 and 3 and a y-intercept of 3, respectively.



c) $y = \dfrac{x^2 + 2x - 8}{x^2 - 2x - 8}$

$= \dfrac{(x-2)(x+4)}{(x-4)(x+2)}, x \neq -2, 4$

The graph will have vertical asymptotes at $x = -2$ and $x = 4$. The graph will have no points of discontinuity. By substituting $y = 0$ and $x = 0$, the graph will have x-intercepts of –4 and 2 and a y-intercept of 1, respectively.



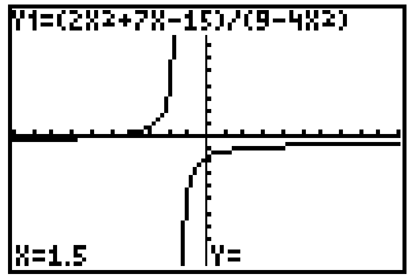
d) $y = \dfrac{2x^2 + 7x - 15}{9 - 4x^2}$

$= \dfrac{(2x-3)(x+5)}{(3-2x)(3+2x)}$

$= \dfrac{(2x-3)(x+5)}{-(2x-3)(2x+3)}$

$= \dfrac{(x+5)}{-(2x+3)}, x \neq -\dfrac{3}{2}, \dfrac{3}{2}$

The graph will have a vertical asymptote at $x = -\dfrac{3}{2}$. Substituting $x = \dfrac{3}{2}$, the graph will have a point of discontinuity at $\left(\dfrac{3}{2}, -\dfrac{13}{12}\right)$, or about (1.5, –1.083). By substituting $y = 0$ and $x = 0$, the graph will have an x-intercept of –5 and a y-intercept of $-\dfrac{5}{3}$, or about –1.67, respectively.



Section 9.2 Page 452 Question 5

a) In factored form, $A(x) = \dfrac{x^2 + 2x}{x^2 + 4}$ is $A(x) = \dfrac{x(x+2)}{x^2 + 4}$. The graph has no vertical asymptotes, no points of discontinuity, and x-intercepts of 0 and –2: graph **C**.

b) In factored form, $B(x) = \dfrac{x-2}{x^2 - 2x}$ is $B(x) = \dfrac{x-2}{x(x-2)}$. The graph has a vertical asymptote at $x = 0$, a point of discontinuity at (2, 0.5), and no x-intercepts: graph **A**.

c) In factored form, $C(x) = \dfrac{x+2}{x^2 - 4}$ is $C(x) = \dfrac{x+2}{(x+2)(x-2)}$. The graph has a vertical asymptote at $x = 2$, a point of discontinuity at (–2, –0.25), and no x-intercepts: graph **D**.

d) In factored form, $D(x) = \dfrac{2x}{x^2 + 2x}$ is $D(x) = \dfrac{2x}{x(x+2)}$. The graph has a vertical asymptote at $x = –2$, a point of discontinuity at (0, 1), and no x-intercepts: graph **B**.

Section 9.2 Page 452 Question 6

a) Since the graph has vertical asymptotes at $x = 1$ and $x = 4$, the denominator has factors $x – 1$ and $x – 4$. Since the graph has x-intercepts of 2 and 3, the numerator has factors $x – 2$ and $x – 3$. Then, the function is of the form

$$y = \frac{(x-2)(x-3)}{(x-1)(x-4)}$$
$$y = \frac{x^2 - 5x + 6}{x^2 - 5x + 4}$$
Choice **C**.

b) Since the graph has vertical asymptotes at $x = –1$ and $x = 2$, the denominator has factors $x + 1$ and $x – 2$. Since the graph has x-intercepts of 1 and 4, the numerator has factors $x – 1$ and $x – 4$. Then, the function is of the form
$$y = \frac{(x-1)(x-4)}{(x+1)(x-2)}$$
$$y = \frac{x^2 - 5x + 4}{x^2 - x - 2}$$
Choice **B**.

c) Since the graph has vertical asymptotes at $x = –2$ and $x = 5$, the denominator has factors $x + 2$ and $x – 5$. Since the graph has x-intercepts of –4 and 3, the numerator has factors $x + 4$ and $x – 3$. Then, the function is of the form
$$y = \frac{(x+4)(x-3)}{(x+2)(x-5)}$$
$$y = \frac{x^2 + x - 12}{x^2 - 3x - 10}$$
Choice **D**.

d) Since the graph has vertical asymptotes at $x = –5$ and $x = 4$, the denominator has factors $x + 5$ and $x – 4$. Since the graph has x-intercepts of –2 and 1, the numerator has factors $x + 2$ and $x – 1$. Then, the function is of the form
$$y = \frac{(x+2)(x-1)}{(x+5)(x-4)}$$
$$y = \frac{x^2 + x - 2}{x^2 + x - 20}$$
Choice **A**.

Section 9.2 Page 453 Question 7

a) Since the graph has a vertical asymptote at $x = –2$, the denominator has factor $x + 2$. Since the graph has an x-intercept of –6, the numerator has factor $x + 6$. Since the graph has a point of discontinuity at (0, 3), both the numerator and denominator have factor x. Then, the function is of the form
$$y = \frac{x(x+6)}{x(x+2)}$$
$$y = \frac{x^2 + 6x}{x^2 + 2x}$$

b) Since the graph has a vertical asymptote at $x = 1$, the denominator has factor $x - 1$. Since the graph has an x-intercept of 7, the numerator has factor $x - 7$. Since the graph has a point of discontinuity at (–3, 2.5), both the numerator and denominator have factor $x + 3$. Then, the function is of the form

$$y = \frac{(x-7)(x+3)}{(x-1)(x+3)}$$

$$y = \frac{x^2 - 4x - 21}{x^2 + 2x - 3}$$

Section 9.2 Page 453 Question 8

a) For vertical asymptotes at $x = \pm 5$ and x-intercepts of –10 and 4,
- the denominator has factors $x + 5$ and $x - 5$
- the numerator has factors $x + 10$ and $x - 4$

Then, the function is of the form $y = \dfrac{(x+10)(x-4)}{(x+5)(x-5)}$.

b) For a vertical asymptote at $x = -4$, a point of discontinuity at $\left(-\dfrac{11}{2}, 9\right)$, and an x-intercept of 8,
- the denominator has factor $x + 4$
- both the numerator and denominator have factor $2x + 11$
- the numerator has factor $x - 8$

Then, the function is of the form $y = \dfrac{(2x+11)(x-8)}{(x+4)(2x+11)}$.

c) For a point of discontinuity at $\left(-2, \dfrac{1}{5}\right)$ a vertical asymptote at $x = 3$, a, and an x-intercept of –1,
- both the numerator and denominator have factor $x + 2$
- the denominator has factor $x - 3$
- the numerator has factor $x + 1$

Then, the function is of the form $y = \dfrac{(x+2)(x+1)}{(x+2)(x-3)}$.

d) For vertical asymptotes at $x = 3$ and $x = \dfrac{6}{7}$, and x-intercepts of $-\dfrac{1}{4}$ and 0,
- the denominator has factors $x - 3$ and $7x - 6$
- the numerator has factors $4x + 1$ and x

Then, the function is of the form $y = \dfrac{(4x+1)x}{(x-3)(7x-6)}$.

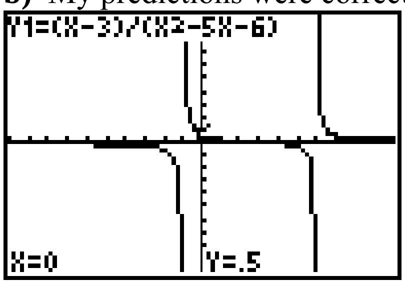
Section 9.2 Page 453 Question 9

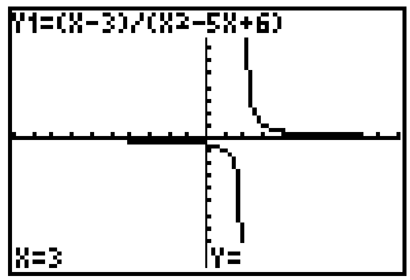
a)
$$f(x) = \frac{x-3}{x^2-5x-6} = \frac{x-3}{(x-6)(x+1)},\ x \neq -1, 6$$

$$g(x) = \frac{x-3}{x^2-5x+6} = \frac{x-3}{(x-2)(x-3)} = \frac{1}{x-2},\ x \neq 2, 3$$

The graphs will be different. The graph of $f(x)$ will have two vertical asymptotes, no points of discontinuity, and one x-intercept. The graph of $g(x)$ will have one vertical asymptote, one point of discontinuity, and no x-intercepts.

b) My predictions were correct.





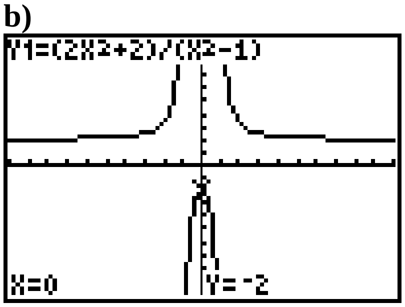
Section 9.2 Page 453 Question 10

Since the graph has points of discontinuity at (–3, –3) and (2, –3), both the numerator and the denominator have factors $x + 3$ and $x - 2$. Then, the function is of the form $y = \frac{-3(x+3)(x-2)}{(x+3)(x-2)}$.

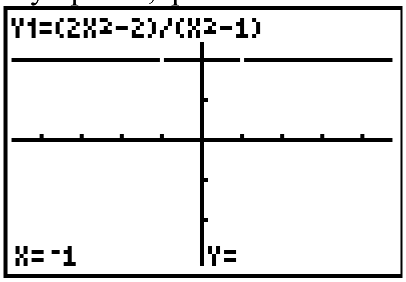
Section 9.2 Page 453 Question 11

a) In factored form, $y = \frac{2x^2+2}{x^2-1}$ is $y = \frac{2(x^2+1)}{(x-1)(x+1)}$. The graph has vertical asymptotes at $x = 1$ and $x = -1$, no points of discontinuity, and no x-intercepts.

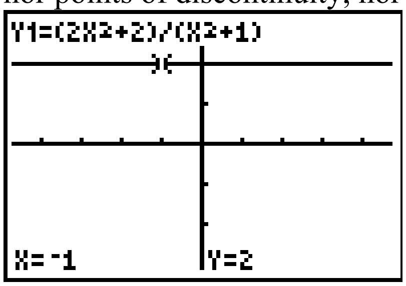
b)



c) i) In factored form, $y = \dfrac{2x^2 - 2}{x^2 - 1}$ is $y = \dfrac{2(x-1)(x+1)}{(x-1)(x+1)}$. The graph has no vertical asymptotes, points of discontinuity at (1, 2) and (–1, 2), and no x-intercepts.



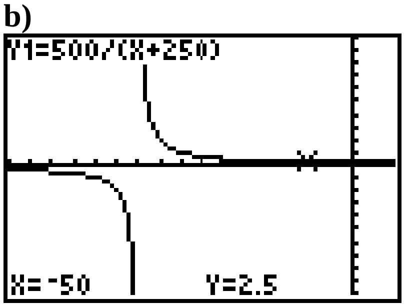
ii) In factored form, $y = \dfrac{2x^2 + 2}{x^2 + 1}$ is $y = \dfrac{2(x^2+1)}{(x^2+1)}$. The graph has no vertical asymptotes, nor points of discontinuity, nor x-intercepts.



Section 9.2 Page 453 Question 12

a) Let w represent the speed of the wind, in kilometres per hour, and t represent the time, in hours, then an equation that represents t as a function of w is $t = \dfrac{500}{250 + w}$, $w \neq -250$.

b)



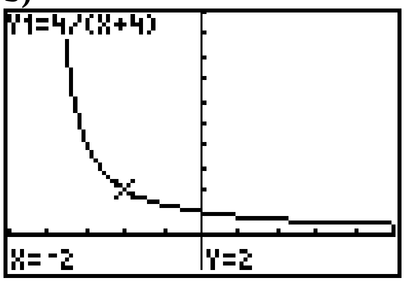
c) When the headwind reaches the speed of the aircraft, theoretically it will come to a standstill, so it will take an infinite amount of time for the aircraft to reach its destination.

d) Example: The realistic part of the graph would be in the range of normal wind speeds for whichever area the aircraft is in.

Section 9.2 Page 454 Question 13

a) The function that relates the time, t, in hours, it will take them to travel 4 km along the channel as a function of the speed, w, in kilometres per hour, of the current is $t = \frac{4}{4+w}$.
The domain in this context is $\{w \mid -4 < w \leq 4, w \in \mathrm{R}\}$.

b)

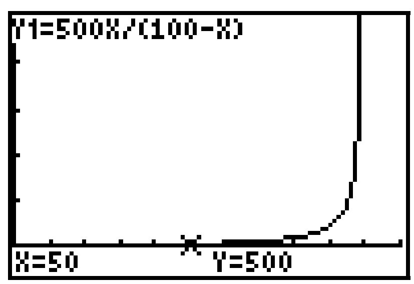


c) When the current against Ryan and Kandra reaches their kayaking speed, theoretically they will come to a standstill, so it will take an infinite amount of time for the them to reach their destination.

Section 9.2 Page 454 Question 14

a) There will be a vertical asymptote at $p = 100$, since $100 - p$ is a factor of the denominator only.

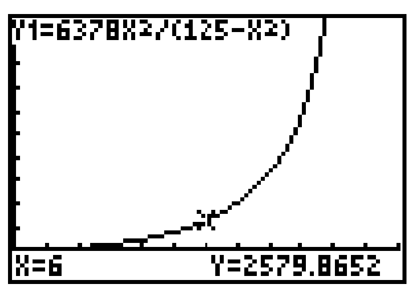
b) The graph shows that the percent of the population vaccinated will never reach 100%.



c) Yes, this is a good model for the estimated cost of vaccinating the population. The vaccination process will become more difficult after the major urban centres have been vaccinated. It will be much more costly to find every single person in the rural areas.

Section 9.2 Page 454 Question 15

a) The portion of the graph applicable is for a domain of $\{v \mid 0 \leq v < \sqrt{125}, v \in \mathrm{R}\}$.



b) As the initial velocity increase, so does the maximum height, but at a greater rate.

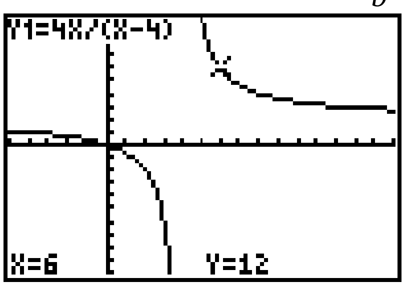
c) There will be a vertical asymptote at $v = \sqrt{125}$, since $\sqrt{125} - v$ is a factor of the denominator only. When the initial velocity reaches the non-permissible value, the object will leave Earth and never return. This models the escape velocity.

Section 9.2 Page 454 Question 16

Since the graph has vertical asymptotes at $x = -2$ and $x = 3$, the denominator has factors $x + 2$ and $x - 3$. Since the graph has x-intercepts of -6 and 2, the numerator has factors $x + 6$ and $x - 2$. Then, the function is of the form $y = \dfrac{(x+6)(x-2)}{(x+2)(x-3)}$.
Check the y-intercept. Substituting $x = 0$, gives a y-intercept of 2. Since the y-intercept of the graph is -1, the function becomes $y = -\dfrac{(x+6)(x-2)}{2(x+2)(x-3)}$.

Section 9.2 Page 455 Question 17

a) Substitute $f = 4$, then $I = \dfrac{4b}{b-4}$.



b) The image distance decreases while the object distance is still less than the focal length. The image distance starts to increase once the object distance is more than the focal length.

c) The non-permissible value results in a vertical asymptote. This relates to the fact that as the object distance approaches the focal length, it gets harder to resolve the image.

Section 9.2 Page 455 Question 18

a) Example: Functions $f(x)$ and $h(x)$ will have similar graphs since they are the same except for a point of discontinuity in the graph of $h(x)$.

b) All three graphs will have a vertical asymptote at $x = -b$, since $x + b$ is a factor of only the denominators. All three graphs will also have an x-intercept of $-a$, since $x + a$ is a factor of only the numerators.

Section 9.2 Page 455 Question 19

Given: $y = \dfrac{x^2 + bx + c}{4x^2 + 29x + c}$ with point of discontinuity at $\left(-8, \dfrac{11}{35}\right)$

From the point of discontinuity, the corresponding factor $x + 8$ is common to both the numerator and the denominator. Substitute $x = -8$ into the corresponding denominator equation and solve the c.

$$\begin{aligned}4x^2 + 29x + c &= 0\\ 4(-8)^2 + 29(-8) + c &= 0\\ c &= -24\end{aligned}$$

Substitute $x = -8$ and $c = -24$ into the corresponding numerator equation and solve for b.

$$\begin{aligned}x^2 + bx + c &= 0\\ (-8)^2 + b(-8) + (-24) &= 0\\ -8b &= -40\\ b &= 5\end{aligned}$$

Then, $y = \dfrac{x^2 + bx + c}{4x^2 + 29x + c}$ becomes $y = \dfrac{x^2 + 5x - 24}{4x^2 + 29x - 24}$. The factored form of $y = \dfrac{x^2 + 5x - 24}{4x^2 + 29x - 24}$ is $y = \dfrac{(x+8)(x-3)}{(x+8)(4x-3)}$. So, the x-intercept is 3 and the vertical asymptote occurs at $x = \dfrac{3}{4}$.

Section 9.2 Page 455 Question 20

For $f(x) = \dfrac{2x^2 - 4x}{x^2 + 3x - 28}$, find $y = \dfrac{1}{4}f(-(x-3))$.

$$f(x) = \frac{2x^2 - 4x}{x^2 + 3x - 28}$$

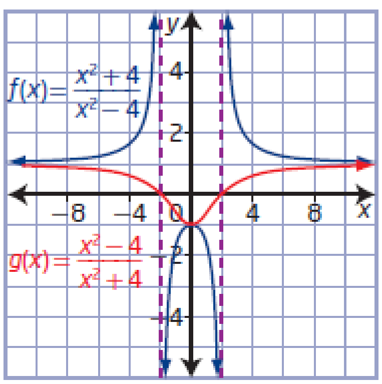
$$\begin{aligned}y &= \frac{1}{4}\left[\frac{2(-(x-3))^2 - 4(-(x-3))}{(-(x-3))^2 + 3(-(x-3)) - 28}\right]\\
&= \frac{1}{4}\left[\frac{2(x^2 - 6x + 9) + 4(x-3)}{x^2 - 6x + 9 - 3(x-3) - 28}\right]\\
&= \frac{1}{4}\left[\frac{2x^2 - 12x + 18 + 4x - 12}{x^2 - 6x + 9 - 3x + 9 - 28}\right]\\
&= \frac{1}{4}\left[\frac{2x^2 - 8x + 6}{x^2 - 9x - 10}\right]\\
&= \frac{1}{2}\left[\frac{x^2 - 4x + 3}{x^2 - 9x - 10}\right]\\
&= \frac{x^2 - 4x + 3}{2x^2 - 18x - 20}\end{aligned}$$

Section 9.2 Page 455 Question 21

a) Since the graph has points of discontinuity at (–4, –2) and (2, 2.5), the corresponding factors $x + 4$ and $x - 2$ are common to both the numerator and the denominator. Ignoring these points, the equation of the linear function is $y = \frac{3}{4}x + 1$ or $y = \frac{1}{4}(3x + 4)$. Putting these together, gives the function $y = \frac{(x+4)(x-2)(3x+4)}{4(x+4)(x-2)}$.

b) Since the graph has points of discontinuity at (–2, 0) and (1, –3), the corresponding factors $x + 2$ and $x - 1$ are common to both the numerator and the denominator. Ignoring these points, the equation of the quadratic function is $y = (x + 2)(x - 2)$. Putting these together, gives the function $y = \frac{(x+2)^2(x-1)(x-2)}{(x+2)(x-1)}$.

Section 9.2 Page 456 Question 22

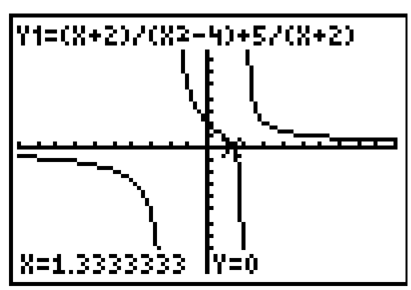


The functions are reciprocals since where the graph of $f(x)$ has vertical asymptotes, the graph of $g(x)$ has x-intercepts.

Section 9.2 Page 456 Question 23

a)
$$\begin{aligned} y &= \frac{x+2}{x^2-4} + \frac{5}{x+2} \\ &= \frac{x+2}{(x+2)(x-2)} + \frac{5}{x+2} \\ &= \frac{x+2+5(x-2)}{(x+2)(x-2)} \\ &= \frac{6x-8}{(x+2)(x-2)}, x \neq \pm 2 \end{aligned}$$

The graph will have vertical asymptotes at $x = –2$ and $x = 2$ and no points of discontinuity.

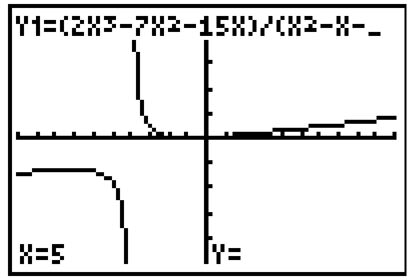


b) $y = \dfrac{2x^3 - 7x^2 - 15x}{x^2 - x - 20}$

$= \dfrac{x(2x+3)(x-5)}{(x-5)(x+4)}$

$= \dfrac{x(2x+3)}{x+4}, x \neq -4, 5$

The graph will have a vertical asymptote at $x = -4$ and a point of discontinuity at $\left(5, \dfrac{65}{9}\right)$.



Section 9.2 Page 456 Question C1

a) Example: No. Some rational functions have no points of discontinuity or asymptotes. For example, $g(x) = \dfrac{x^2 - 4}{x^2 + 4}$ and $h(x) = \dfrac{x^2 + 2x}{x^2 + 4}$ have no vertical asymptotes nor points of discontinuity since there are no values that make the denominator 0 nor factors common between the numerator and denominator.

b) Example: A rational function is a function that has a polynomial in the numerator and/or in the denominator. A factor of only the denominator corresponds to a vertical asymptote. A factor of both the numerator and the denominator corresponds to a point of discontinuity.

Section 9.2 Page 456 Question C2

Example: True. All polynomial functions are rational functions since it is possible to express a polynomial function as a rational function with a denominator of 1.

Section 9.2 Page 456 Question C3

Example: The graphs of rational functions in Section 9.1 all had asymptotes and two branches. The graphs of rational functions in Section 9.2 had

- no branches, two branches, or three branches
- asymptotes only
- points of discontinuity only
- asymptotes and points of discontinuity
- neither asymptotes nor points of discontinuity

Section 9.3 Connecting Graphs and Rational Equations

Section 9.3 Page 465 Question 1

a) The single function that can be used to solve $\frac{x}{x-2}+6=x$ is $y=\frac{x}{x-2}-x+6$: choice **B**.

b) The single function that can be used to solve $6-x=\frac{x}{x-2}+2$ is $y=\frac{x}{x-2}+x-4$: choice **D**.

c) The single function that can be used to solve $6-\frac{x}{x-2}=x-2$ is $y=\frac{x}{x-2}+x-8$: choice **A**.

d) The single function that can be used to solve $x+6=\frac{x}{x-2}$ is $y=\frac{x}{x-2}-x-6$: choice **C**.

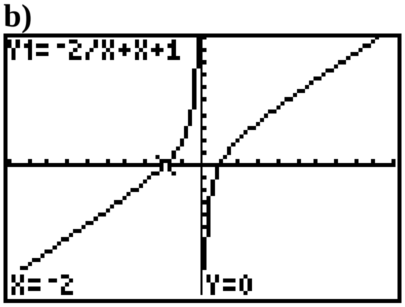
Section 9.3 Page 465 Question 2

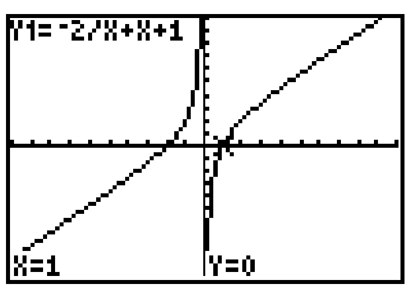
a) The equation has a non-permissible value of 0.

$$
\begin{aligned}
-\frac{2}{x}+x+1&=0\\
-2+x^2+x&=0\\
x^2+x-2&=0\\
(x+2)(x-1)&=0\\
x=-2 \text{ or } x&=1
\end{aligned}
$$

By inspection, both values check. The equation has two solutions, $x = –2$ and $x = 1$.

b)





The equation has two solutions, $x = –2$ and $x = 1$.

c) The value of the function is 0 when the value of x is –2 or 1. The x-intercepts of the graph of the corresponding function are the same as the roots of the equation.

Section 9.3 Page 465 Question 3

a) The equation has a non-permissible value of $-\frac{4}{3}$.

$$\begin{aligned} \frac{5x}{3x+4} &= 7 \\ 5x &= 7(3x+4) \\ 5x &= 21x+28 \\ -16x &= 28 \\ x &= -\frac{28}{16} \\ x &= -\frac{7}{4} \end{aligned}$$

b) The equation has a non-permissible value of 0.

$$\begin{aligned} 2 &= \frac{20-3x}{x} \\ 2x &= 20-3x \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

c) The equation has a non-permissible value of 2.

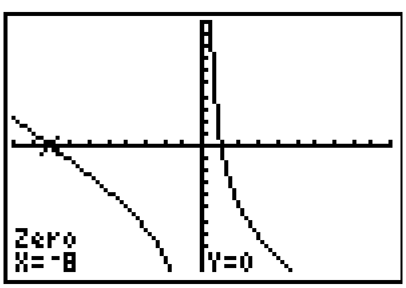
$$\begin{aligned} \frac{x^2}{x-2} &= x-6 \\ x^2 &= (x-6)(x-2) \\ x^2 &= x^2-8x+12 \\ 8x &= 12 \\ x &= \frac{12}{8} \\ x &= \frac{3}{2} \end{aligned}$$

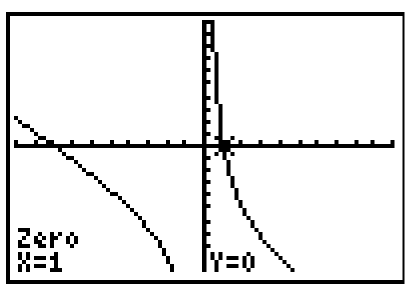
d) The equation has non-permissible values of –3 and 0.

$$\begin{aligned} 1+\frac{2}{x} &= \frac{x}{x+3} \\ \frac{x+2}{x} &= \frac{x}{x+3} \\ x^2 &= (x+2)(x+3) \\ x^2 &= x^2+5x+6 \\ -5x &= 6 \\ x &= -\frac{6}{5} \end{aligned}$$

Section 9.3 Page 465 Question 4

a) Graph the single function $y = \frac{8}{x} - x - 7$ and identify the x-intercepts.





The equation has two solutions, $x = –8$ and $x = 1$.
Check:

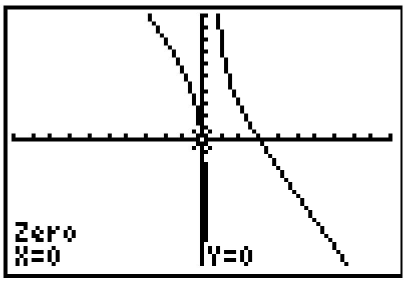
For $x = -8$,

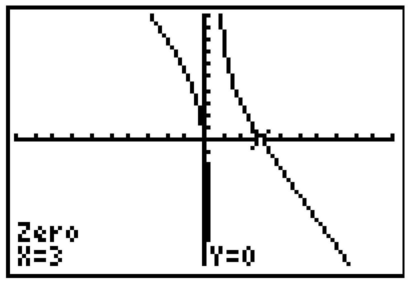
Left Side	Right Side
$\frac{8}{x} - 4$	$x + 3$
$= \frac{8}{-8} - 4$	$= -8 + 3$
$= -5$	$= -5$

For $x = 1$,

Left Side	Right Side
$\frac{8}{x} - 4$	$x + 3$
$= \frac{8}{1} - 4$	$= 1 + 3$
$= 4$	$= 4$

b) Graph the single function $y = \frac{10x}{2x-1} - 2x$ and identify the x-intercepts.





The equation has two solutions, $x = 0$ and $x = 3$.

Check:

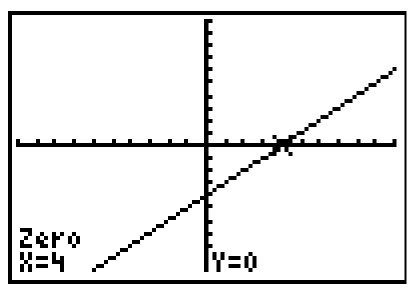
For $x = 0$,

Left Side	Right Side
$2x$	$\frac{10x}{2x-1}$
$= 2(0)$	$= \frac{10(0)}{2(0)-1}$
$= 0$	$= 0$

For $x = 3$,

Left Side	Right Side
$2x$	$\frac{10x}{2x-1}$
$= 2(3)$	$= \frac{10(3)}{2(3)-1}$
$= 6$	$= 6$

c) Graph the single function $y = \frac{3x^2 + 4x - 15}{x+3} - 2x + 1$ and identify the x-intercepts.



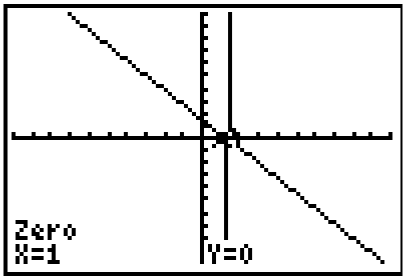
The equation has one solution, $x = 4$.

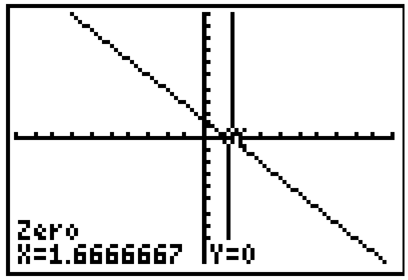
Check:

For $x = 4$,

Left Side	Right Side
$\frac{3x^2 + 4x - 15}{x+3}$	$2x - 1$
$= \frac{3(4)^2 + 4(4) - 15}{4+3}$	$= 2(4) - 1$
$= 7$	$= 7$

d) Graph the single function $y = 1 + \frac{x^2 - 4x}{7 - 5x} - x - \frac{3}{5x - 7}$ and identify the x-intercepts.





The equation has two solutions, $x = 1$ and $x = \frac{5}{3}$.

Check:

For $x = 1$,

Left Side

$$\frac{3}{5x-7} + x$$
$$= \frac{3}{5(1) - 7} + 1$$
$$= -\frac{1}{2}$$

Right Side

$$1 + \frac{x^2 - 4x}{7 - 5x}$$
$$= 1 + \frac{1^2 - 4(1)}{7 - 5(1)}$$
$$= -\frac{1}{2}$$

For $x = \frac{5}{3}$,

Left Side

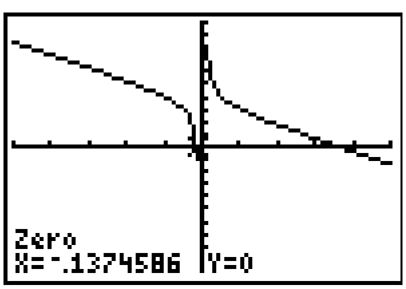
$$\frac{3}{5x-7} + x$$
$$= \frac{3}{5\left(\frac{5}{3}\right) - 7} + \frac{5}{3}$$
$$= \frac{47}{12}$$

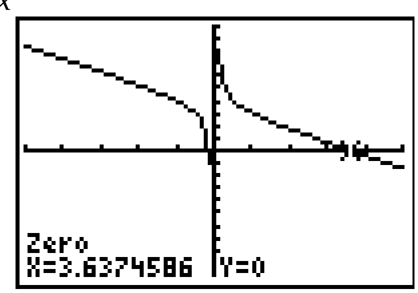
Right Side

$$1 + \frac{x^2 - 4x}{7 - 5x}$$
$$= 1 + \frac{\left(\frac{5}{3}\right)^2 - 4\left(\frac{5}{3}\right)}{7 - 5\left(\frac{5}{3}\right)}$$
$$= \frac{47}{12}$$

Section 9.3 Page 466 Question 5

a) Graph the single function $y = \frac{x+1}{2x} - x + 3$ and identify the x-intercepts.

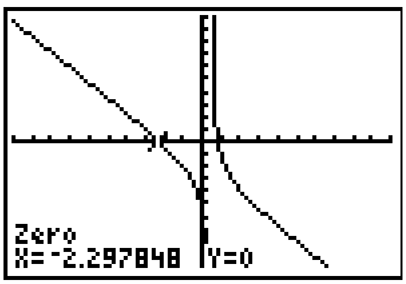


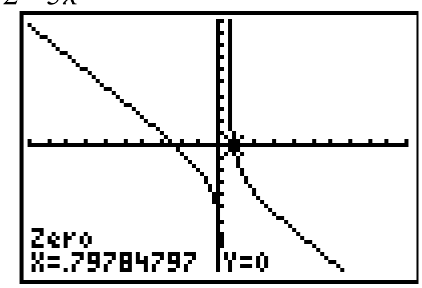


The equation has two solutions, $x \approx -0.14$ and $x \approx 3.64$.

$$\frac{x+1}{2x} = x-3$$
$$x+1 = 2x(x-3)$$
$$x+1 = 2x^2 - 6x$$
$$0 = 2x^2 - 7x - 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-7 \pm \sqrt{57}}{4}$$
$x \approx -0.14$ or $x \approx 3.64$

b) Graph the single function $y = \frac{x^2 - 4x - 5}{2 - 5x} - x - 3$ and identify the x-intercepts.

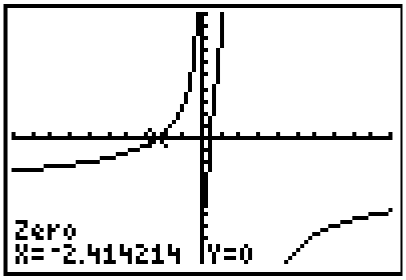


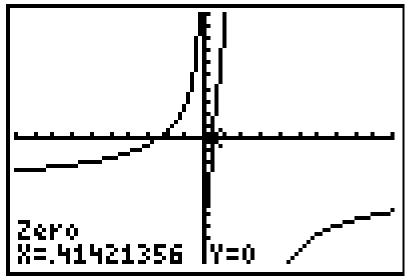


The equation has two solutions, $x \approx -2.30$ and $x \approx 0.80$.

$$\frac{x^2 - 4x - 5}{2 - 5x} = x + 3$$
$$x^2 - 4x - 5 = (x+3)(2-5x)$$
$$x^2 - 4x - 5 = -5x^2 - 13x + 6$$
$$6x^2 + 9x - 11 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(6)(-11)}}{2(6)}$$
$$= \frac{-9 \pm \sqrt{345}}{12}$$
$x \approx -2.30$ or $x \approx 0.80$

c) Graph the single function $y = 3 - \frac{7x}{x-2} - \frac{2}{x}$ and identify the x-intercepts.





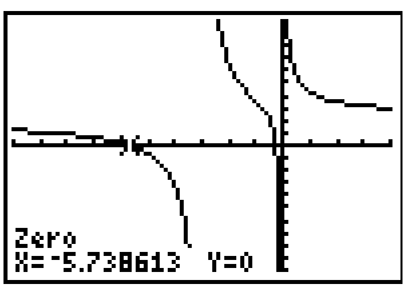
The equation has two solutions, $x \approx -2.41$ and $x \approx 0.41$.

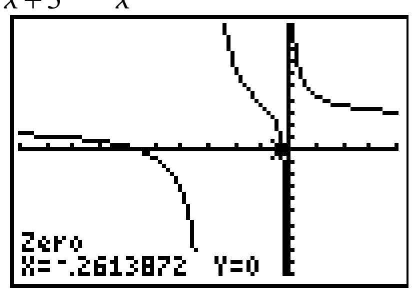
$$\begin{aligned}
\frac{2}{x} &= 3 - \frac{7x}{x-2} \\
\frac{2}{x} &= \frac{-6-4x}{x-2} \\
2(x-2) &= x(-6-4x) \\
2x - 4 &= -6x - 4x^2
\end{aligned}$$

$$\begin{aligned}
4x^2 + 8x - 4 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-8 \pm \sqrt{8^2 - 4(4)(-4)}}{2(4)} \\
&= \frac{-8 \pm \sqrt{128}}{8}
\end{aligned}$$

$x \approx -2.41$ or $x \approx 0.41$

d) Graph the single function $y = 1 + \frac{5}{x+3} + \frac{x+1}{x}$ and identify the x-intercepts.





The equation has two solutions, $x \approx -5.74$ and $x \approx -0.26$.

$$\begin{aligned} 2+\frac{5}{x+3} &= 1-\frac{x+1}{x} \\ \frac{2x+11}{x+3} &= \frac{-1}{x} \\ x(2x+11) &= -(x+3) \\ 2x^2+11x &= -x-3 \end{aligned}$$

$$2x^2+12x+3=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-12\pm\sqrt{12^2-4(2)(3)}}{2(2)}$$

$$=\frac{-12\pm\sqrt{120}}{4}$$

$x \approx –5.74$ or $x = –0.26$

Section 9.3 Page 466 Question 6

a) The equation has a non-permissible value of 2.

$$\begin{aligned} \frac{3x}{x-2}+5x &= \frac{x+4}{x-2} \\ \frac{5x^2-7x}{x-2} &= \frac{x+4}{x-2} \\ 5x^2-7x &= x+4 \end{aligned}$$

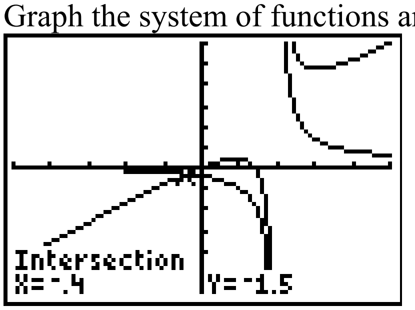
$$5x^2-8x-4=0$$

$$(5x+2)(x-2)=0$$

$x=-\frac{2}{5}$ or $x = 2$

The solution is $x=-\frac{2}{5}$, since $x = 2$ is an extraneous root.

Graph the system of functions and find the points of intersection.

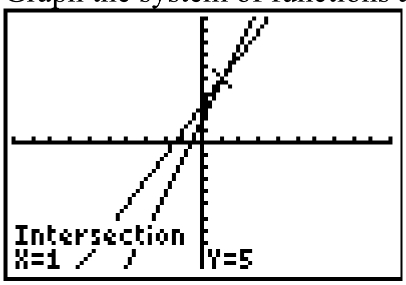


The solution is $x = –0.4$, or $-\frac{2}{5}$, which is the same as the answer found algebraically.

b) The equation has a non-permissible value of –4.

$$2x+3=\frac{3x^2+14x+8}{x+4}$$
$$(2x+3)(x+4)=3x^2+14x+8$$
$$2x^2+11x+12=3x^2+14x+8$$
$$0=x^2+3x-4$$
$$0=(x+4)(x-1)$$

$x = -4$ or $x = 1$
The solution is $x = 1$, since $x = -4$ is an extraneous root.
Graph the system of functions and find the points of intersection.

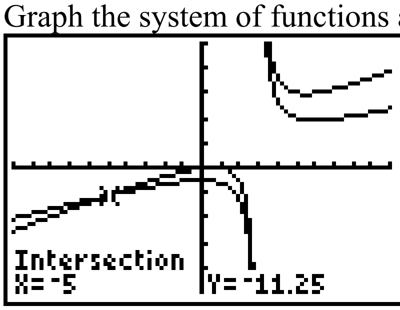


The solution is $x = 1$, which is the same as the answer found algebraically.

c) The equation has a non-permissible value of 3.

$$\frac{6x}{x-3}+3x=\frac{2x^2}{x-3}-5$$
$$\frac{3x^2-3x}{x-3}=\frac{2x^2-5x+15}{x-3}$$
$$3x^2-3x=2x^2-5x+15$$
$$x^2+2x-15=0$$
$$(x+5)(x-3)=0$$

$x = -5$ or $x = 3$
The solution is $x = -5$, since $x = 3$ is an extraneous root.
Graph the system of functions and find the points of intersection.



The solution is $x = -5$, which is the same as the answer found algebraically.

d) The equation has non-permissible values of 0 and 1.

$$\frac{2x-1}{x^2-x}+4=\frac{x}{x-1}$$

$$\frac{4x^2-2x-1}{x(x-1)}=\frac{x}{x-1}$$

$$4x^2-2x-1=x^2$$

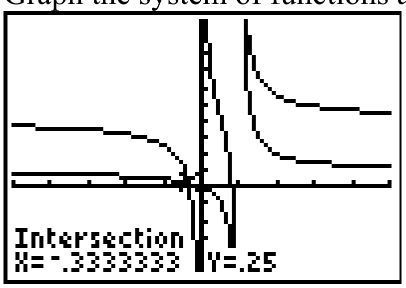
$$3x^2-2x-1=0$$

$$(3x+1)(x-1)=0$$

$$x=-\frac{1}{3} \text{ or } x=1$$

The solution is $x=-\frac{1}{3}$, since $x = 1$ is an extraneous root.

Graph the system of functions and find the points of intersection.



The solution is $x=-\frac{1}{3}$, which is the same as the answer found algebraically.

Section 9.3 Page 466 Question 7

Example: Yunah's approach is correct, but she should have indicated that the non-permissible value is 1.

Section 9.3 Page 466 Question 8

Solve algebraically. The equation has non-permissible values of 1 and –2.

$$\frac{2x+1}{x-1}=\frac{2}{x+2}-\frac{3}{2}$$

$$\frac{2x+1}{x-1}=\frac{4-(3x+6)}{2(x+2)}$$

$$2(x+2)(2x+1)=(x-1)(-3x-2)$$

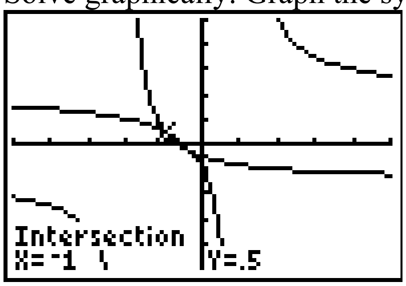
$$4x^2+10x+4=-3x^2+x+2$$

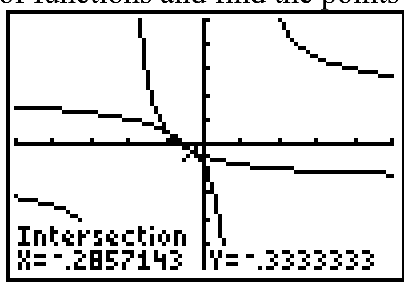
$$7x^2+9x+2=0$$

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-9 \pm \sqrt{9^2 - 4(7)(2)}}{2(7)} \\
&= \frac{-9 \pm \sqrt{25}}{14} \\
&= \frac{-9 \pm 5}{14}
\end{aligned}
$$

The solution is $x = -\frac{2}{7}$, or about –0.29, and $x = -1$.

Solve graphically. Graph the system of functions and find the points of intersection.





The solution is $x = -1$ and $x \approx -0.29$, which is the same as the solution found algebraically.

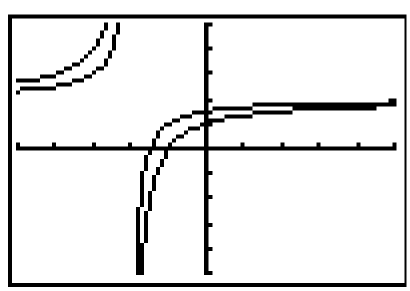
Section 9.3 Page 466 Question 9

Solve algebraically. The equation has non-permissible value is –2.

$$
\begin{aligned}
2 - \frac{1}{x+2} &= \frac{x}{x+2} + 1 \\
1 &= \frac{x}{x+2} + \frac{1}{x+2} \\
1 &= \frac{x+1}{x+2}
\end{aligned}
$$

There is no solution.

Solve graphically. Graph the system of functions and find the points of intersection.



The graphs do not intersect, so there is no solution. This is the same as the result found algebraically.

Section 9.3 Page 466 Question 10

Substitute $P = 500$ and $I = 5$ into $I = \dfrac{P}{4\pi d^2}$.

$$\begin{aligned} I &= \frac{P}{4\pi d^2} \\ {\color{red}5} &= \frac{{\color{red}500}}{4\pi d^2} \\ d^2 &= \frac{500}{20\pi} \\ d^2 &= \frac{25}{\pi} \\ d &= \frac{5}{\sqrt{\pi}} \\ d &= 2.820... \end{aligned}$$

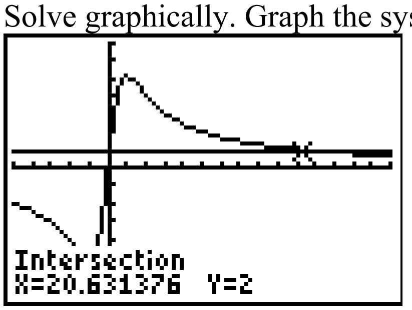
At approximately 2.82 m from a 500-W light source the is intensity 5 W/m^2.

Section 9.3 Page 466 Question 11

Substitute $C = 2$ into $C(t) = \dfrac{50t}{1.2t^2 + 5}$.

$$\begin{aligned} C(t) &= \frac{50t}{1.2t^2 + 5} \\ {\color{red}2} &= \frac{50t}{1.2t^2 + 5} \end{aligned}$$

Solve graphically. Graph the system of functions and find the points of intersection.



Approximately 20.6 h after drinking coffee the person's level dropped to 2 mg/L.

Section 9.3 Page 467 Question 12

Substitute $T = 10$ and $a = 30$ into $T = \dfrac{ab}{a+b}$.

$$T = \frac{ab}{a+b}$$
$$10 = \frac{30b}{30+b}$$
$$10(30+b) = 30b$$
$$300 + 10b = 30b$$
$$300 = 20b$$
$$b = 15$$
It would take James 15 min to set it up by himself.

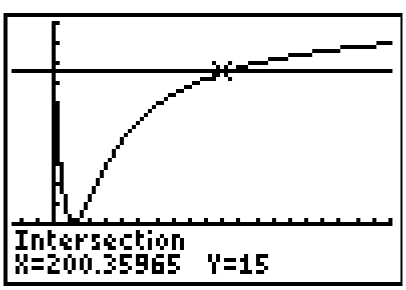
Section 9.3 Page 467 Question 13

a) Given: 28 shots on net, scored 2 goals
If x represents the number of shots she takes from now on and she scores on half of them, then a function that represents Rachel's shooting percentage is $p = \dfrac{2+0.5x}{28+x}$.

b) Substitute $p = 0.3$.
$$p = \frac{2+0.5x}{28+x}$$
$$0.3 = \frac{2+0.5x}{28+x}$$
$$0.3(28+x) = 2+0.5x$$
$$6.4 = 0.2x$$
$$x = 32$$
It will take 32 more shots for her to bring her shooting percentage up to her target.

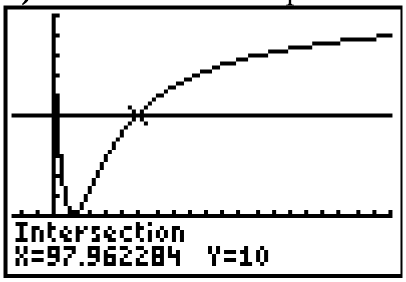
Section 9.3 Page 467 Question 14

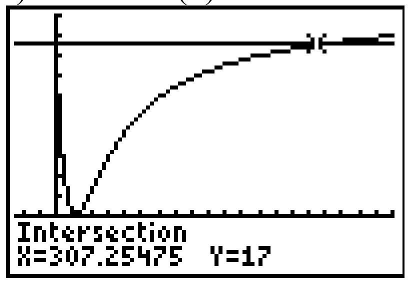
a) Substitute $C(T) = 15$ into $C(T) = \dfrac{21.2T^2 - 877T + 9150}{T^2 - 23.6T + 760}$. Solve graphically.



According to this model, at a temperature of approximately 200.4 K the coefficient of thermal expansion will be 15.

b) Determine the temperature for $C(T) = 10$ and $C(T) = 17$.





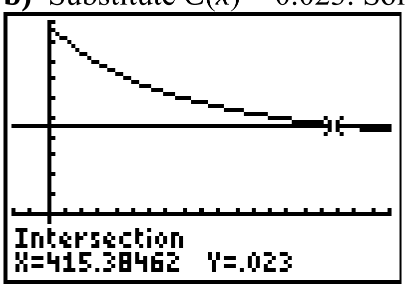
The temperature has to increase about 307.3 – 98.0, or 209.3 K.

Section 9.3 Page 467 Question 15

a) Given: solution A 0.05 g/mL, solution B 0.01 g/mL, start with 200 mL of solution A and pour in x millilitres of solution B

An equation for the concentration, $C(x)$, of the solution after x millilitres have been added is $C(x) = \dfrac{200(0.05) + 0.01x}{200 + x}$ or $C(x) = \dfrac{10 + 0.01x}{200 + x}$.

b) Substitute $C(x) = 0.023$. Solve graphically.

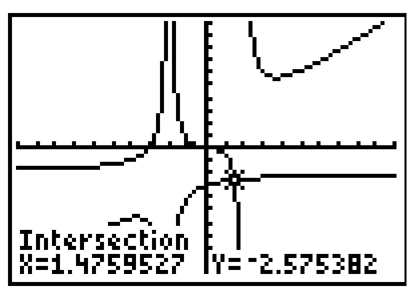


Approximately 415 mL need to be added to make a solution with a concentration of 0.023 g/mL.

Section 9.3 Page 467 Question 16

Solve graphically. Graph the system of functions and find the points of intersection.

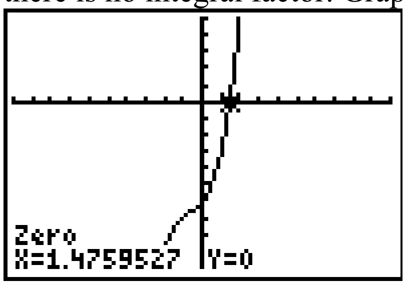
The solution is $x \approx 1.48$.



Solve algebraically. The equation has non-permissible values of –2 and 2.

$$\frac{x}{x+2} - 3 = \frac{5x}{x^2-4} + x$$
$$x(x-2) - 3(x^2-4) = 5x + x(x^2-4)$$
$$x^2 - 2x - 3x^2 + 12 = 5x + x^3 - 4x$$
$$0 = x^3 + 2x^2 + 3x - 12$$

Using the factor theorem to test the possible integral factors of ±1, ±2, ±3, ±4, ±6, ±12, there is no integral factor. Graph $y = x^3 + 2x^2 + 3x - 12$ and determine the x-intercept.

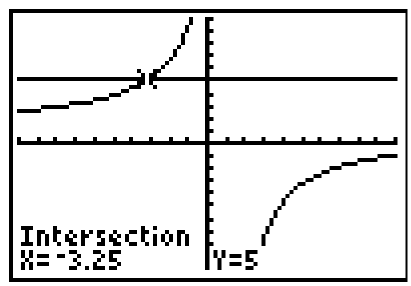


The solution is $x \approx 1.48$.

Section 9.3 Page 467 Question 17

a) $\frac{x-18}{x-1} \le 5$

$$x - 18 \le 5(x-1)$$
$$x - 18 \le 5x - 5$$
$$-4x \le 13$$
$$x \ge -\frac{13}{4}$$

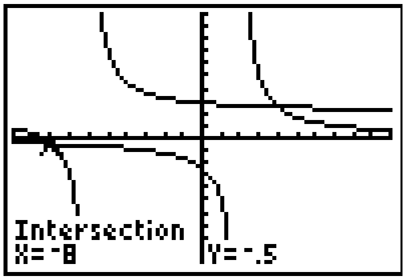


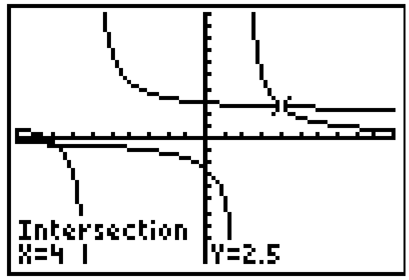
Taking into consideration the non-permissible value of $x = 1$, the solution is $x \le -\frac{13}{4}$ or $x > 1$.

b) $\frac{5}{x-2} \ge \frac{2x+17}{x+6}$

$$5(x+6) \ge (x-2)(2x+17)$$
$$5x + 30 \ge 2x^2 + 13x - 34$$
$$0 \ge 2x^2 + 8x - 64$$
$$0 \ge x^2 + 4x - 32$$
$$0 \ge (x+8)(x-4)$$

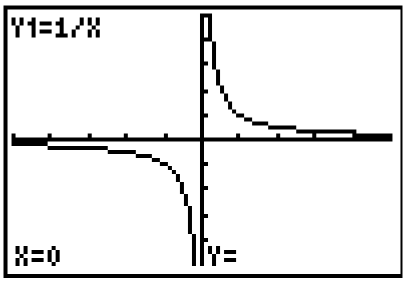
Taking into consideration the non-permissible values of $x = 2$ and $x = -6$, the solution is $-8 \le x < -6$ or $2 < x \le 4$.





Section 9.3 Page 467 Question C1

Example: No, this is incorrect. For example, $\frac{1}{x} = 0$ has no solution.



Section 9.3 Page 467 Question C2

Example: The extraneous root in the radical equation $x = \sqrt{x+6}$ occurs because there is a restriction that the radicand be positive. This same principle of restricted domain is the reason why the rational equation $\frac{3x}{x+2} = x - \frac{6}{x+2}$ has an extraneous root. Extraneous roots can be identified by comparing any solutions with the domain restrictions.

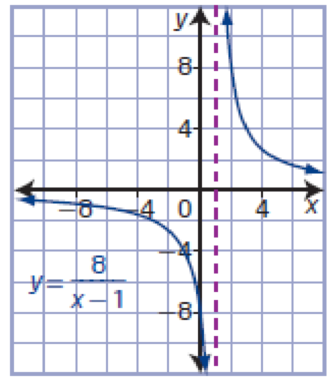
Section 9.3 Page 467 Question C3

Example: I prefer a combination of graphical and algebraic to solve rational equations. The graphical method eliminates any extraneous solutions, but may give only approximate answers. The algebraic method may result in extraneous solutions but can provide exact answers.

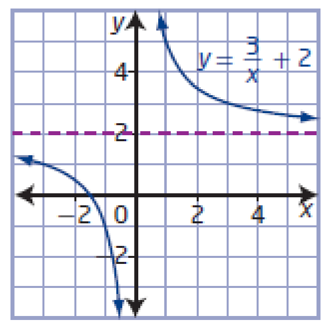
Chapter 9 Review

Chapter 9 Review Page 468 Question 1

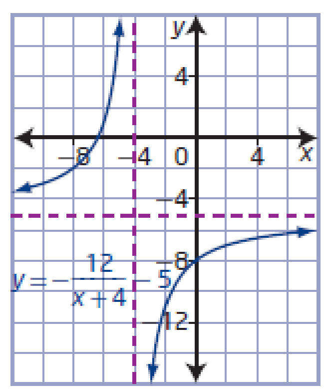
a) For $y = \frac{8}{x-1}$, $a = 8$, $h = 1$, and $k = 0$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 8 and translated 1 unit to the right. The domain is $\{x \mid x \neq 1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$. There is no x-intercept. The y-intercept is –8. The vertical asymptote is at $x = 1$ and the horizontal asymptote is at $y = 0$.



b) For $y = \frac{3}{x} + 2$, $a = 3$, $h = 0$, and $k = 2$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 3 and translated 2 units up. The domain is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 2, y \in \mathrm{R}\}$. The x-intercept is $-\frac{3}{2}$. There is no y-intercept. The vertical asymptote is at $x = 0$ and the horizontal asymptote is at $y = 2$.

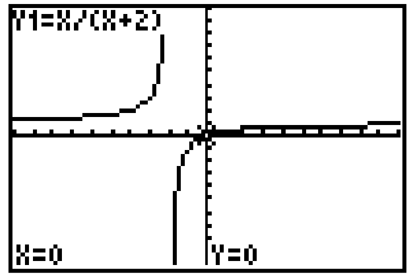


c) For $y = -\frac{12}{x+4} - 5$, $a = –12$, $h = –4$, and $k = –5$. The graph of the base function $y = \frac{1}{x}$ must be stretched vertically by a factor of 12, reflected in the x-axis, and translated 4 units to the left and 5 units down. The domain is $\{x \mid x \neq -4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -5, y \in \mathrm{R}\}$. The x-intercept is $-\frac{32}{5}$, or –6.4. The y-intercept is –8. The vertical asymptote is at $x = –4$ and the horizontal asymptote is at $y = –5$.

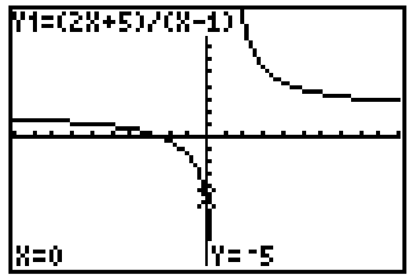


Chapter 9 Review Page 468 Question 2

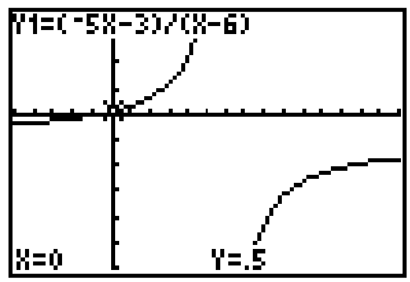
a) For $y = \dfrac{x}{x+2}$, the vertical asymptote is at $x = -2$, the horizontal asymptote is at $y = 1$, the x-intercept is 0, and the y-intercept is 0.



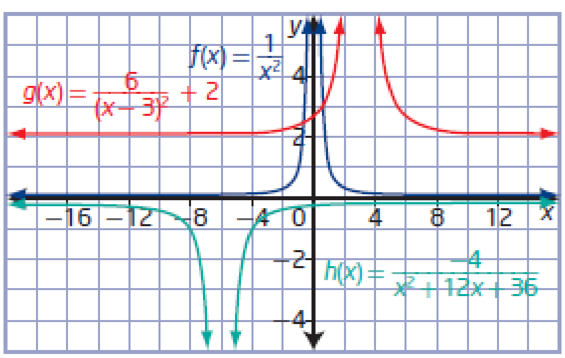
b) For $y = \dfrac{2x+5}{x-1}$, the vertical asymptote is at $x = 1$, the horizontal asymptote is at $y = 2$, the x-intercept is –2.5, and the y-intercept is –5.



c) For $y = \dfrac{-5x-3}{x-6}$, the vertical asymptote is at $x = 6$, the horizontal asymptote is at $y = -5$, the x-intercept is –0.6, and the y-intercept is 0.5.



Chapter 9 Review Page 468 Question 3



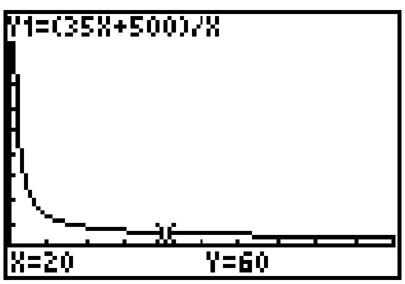
Characteristic	$f(x) = \dfrac{1}{x^2}$	$g(x) = \dfrac{6}{(x-3)^2} + 2$	$h(x) = \dfrac{-4}{x^2+12x+36}$
Non-permissible value	$x = 0$	$x = 3$	$x = -6$

Behaviour near non-permissible value	As x approaches 0, $\lvert y \rvert$ becomes very large.	As x approaches 3, $\lvert y \rvert$ becomes very large.	As x approaches −6, $\lvert y \rvert$ becomes very large.
End behaviour	As $\lvert x \rvert$ becomes very large, y approaches 0.	As $\lvert x \rvert$ becomes very large, y approaches 2.	As $\lvert x \rvert$ becomes very large, y approaches 0.
Domain	$\{x \mid x \neq 0, x \in \mathrm{R}\}$	$\{x \mid x \neq 3, x \in \mathrm{R}\}$	$\{x \mid x \neq -6, x \in \mathrm{R}\}$
Range	$\{y \mid y > 0, y \in \mathrm{R}\}$	$\{y \mid y > 2, y \in \mathrm{R}\}$	$\{y \mid y < 0, y \in \mathrm{R}\}$
Equation of vertical asymptote	$x = 0$	$x = 3$	$x = -6$
Equation of horizontal asymptote	$y = 0$	$y = 2$	$y = 0$

Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consists of all real numbers except for a single value. The range of each function consists of a restricted set of the real numbers. $\lvert y \rvert$ becomes very large for each function when the values of x approach the non-permissible value for the function.

Chapter 9 Review Page 468 Question 4

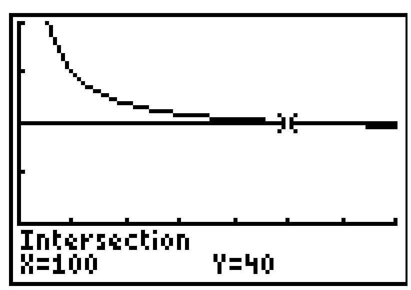
a) Let x represent the number of uniforms ordered. Let y represent the average cost per uniform. Then, a function to model this situation is $y = \dfrac{500 + 35x}{x}$.



b) As the number of uniforms ordered increases, the average cost per uniform decreases.

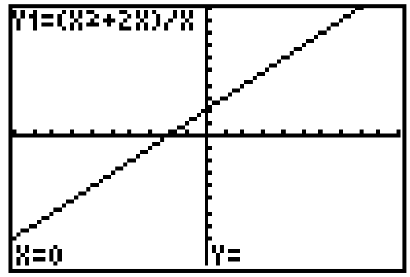
c) Graph $y = \dfrac{500 + 35x}{x}$ and $y = 40$, then determine the point of intersection.

The league needs to order 100 uniforms.

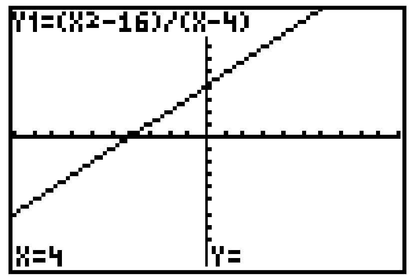


Chapter 9 Review Page 468 Question 5

a) The function $y = \dfrac{x^2 + 2x}{x}$ can be written in factored form as $y = \dfrac{x(x+2)}{x}$. The graph has a point of discontinuity at (0, 2), since x is a factor of both the numerator and the denominator.



b) The function $y = \dfrac{x^2 - 16}{x - 4}$ can be written in factored form as $y = \dfrac{(x-4)(x+4)}{x-4}$. The graph has a point of discontinuity at (4, 8), since $x - 4$ is a factor of both the numerator and the denominator.



c) The function $y = \dfrac{2x^2 - 3x - 5}{2x - 5}$ can be written in factored form as $y = \dfrac{(2x-5)(x+1)}{2x-5}$. The graph has a point of discontinuity at (2.5, 3.5), since $2x - 5$ is a factor of both the numerator and the denominator.

Chapter 9 Review Page 468 Question 6

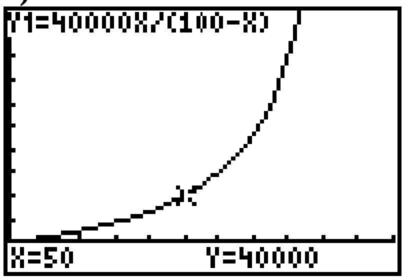
In factored form, $A(x) = \dfrac{x-4}{x^2 - 5x + 4}$ is $A(x) = \dfrac{x-4}{(x-4)(x-1)}$. The graph has a vertical asymptote at $x = 1$, a point of discontinuity at about (4, 0.3), and no x-intercepts: **Graph 3**.

In factored form, $B(x) = \dfrac{x^2 + 5x + 4}{x^2 + 1}$ is $B(x) = \dfrac{(x+4)(x+1)}{x^2 + 1}$. The graph has no vertical asymptotes, no points of discontinuity, and x-intercepts of –4 and –1: **Graph 1**.

In factored form, $C(x) = \dfrac{x-1}{x^2 - 4}$ is $C(x) = \dfrac{x-1}{(x+2)(x-2)}$. The graph has vertical asymptote at $x = –2$ and $x = 2$, no points of discontinuity, and an x-intercept of 1: **Graph 2**.

Chapter 9 Review Page 469 Question 7

a)



b) As the percent of the spill cleaned up approaches 100, the cost approaches infinity.

c) No, it is not possible to clean up 100% of a spill. The function has a vertical asymptote at $p = 100$.

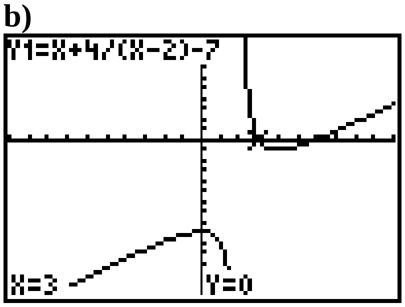
Chapter 9 Review Page 469 Question 8

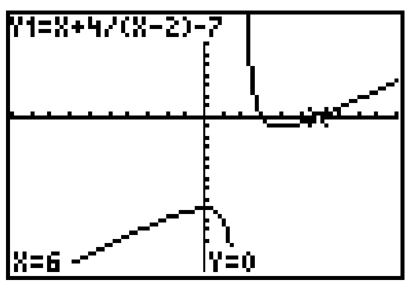
a) The equation has a non-permissible value of 2.

$$x + \frac{4}{x-2} - 7 = 0$$
$$x(x-2) + 4 - 7(x-2) = 0$$
$$x^2 - 9x + 18 = 0$$
$$(x-3)(x-6) = 0$$
$$x = 3 \text{ or } x = 6$$

By inspection, both values check. The equation has two solutions, $x = 3$ and $x = 6$.

b)



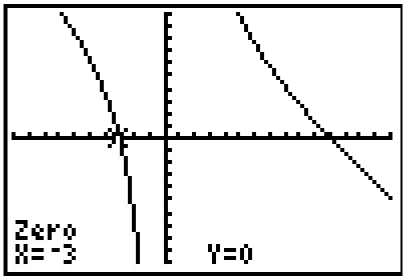


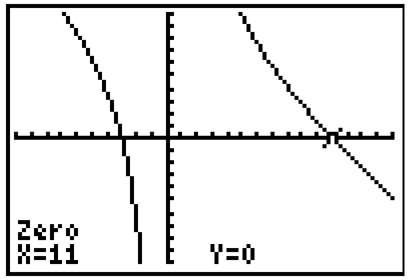
The equation has two solutions, $x = 3$ and $x = 6$.

c) The value of the function is 0 when the value of x is 3 or 6. The x-intercepts of the graph of the corresponding function are the same as the roots of the equation.

Chapter 9 Review   Page 469   Question 9

a) Graph the single function $y = \dfrac{33}{x} - x + 8$ and identify the *x*-intercepts.

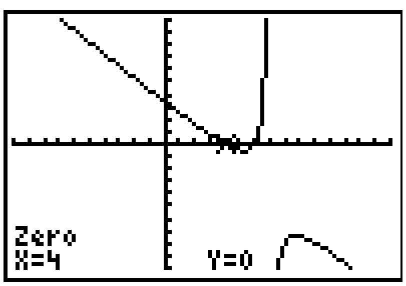


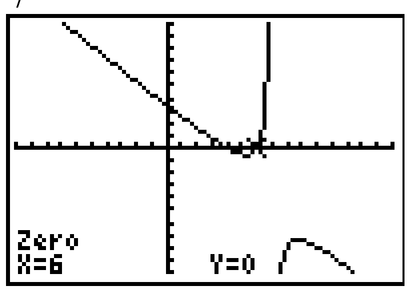


The equation has two solutions, $x = -3$ and $x = 11$.

$$\begin{aligned} x - 8 &= \frac{33}{x} \\ x(x-8) &= 33 \\ x^2 - 8x &= 33 \\ x^2 - 8x - 33 &= 0 \\ (x-11)(x+3) &= 0 \end{aligned}$$
$x = -3$ or $x = 11$

b) Graph the single function $y = \dfrac{x-10}{x-7} - x + 2$ and identify the *x*-intercepts.

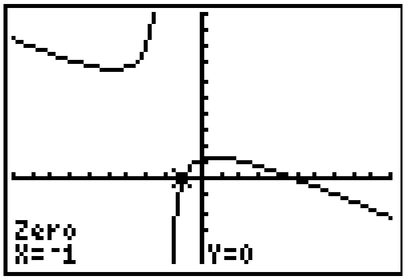


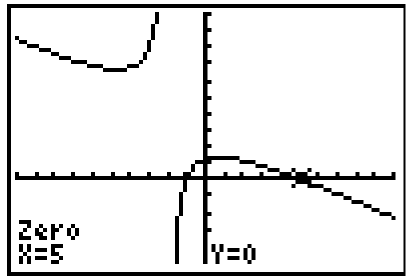


The equation has two solutions, $x = 4$ and $x = 6$.

$$\begin{aligned} \frac{x-10}{x-7} &= x - 2 \\ x - 10 &= (x-2)(x-7) \\ x - 10 &= x^2 - 9x + 14 \\ 0 &= x^2 - 10x + 24 \\ 0 &= (x-6)(x-4) \end{aligned}$$
$x = 6$ or $x = 4$

c) Graph the single function $y = \frac{3x-1}{x+2} - x + 3$ and identify the x-intercepts.



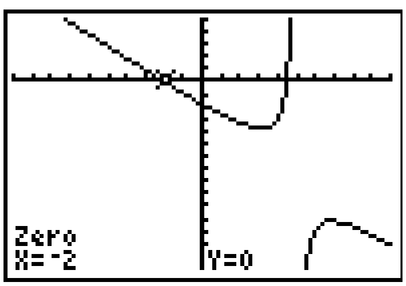


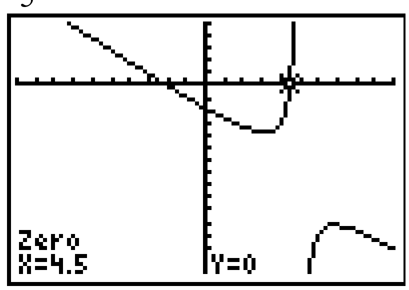
The equation has two solutions, $x = -1$ and $x = 5$.

$$
\begin{aligned}
x &= \frac{3x-1}{x+2} + 3 \\
x(x+2) &= 3x - 1 + 3(x+2) \\
x^2 + 2x &= 3x - 1 + 3x + 6 \\
x^2 - 4x - 5 &= 0 \\
(x-5)(x+1) &= 0
\end{aligned}
$$

$x = 5$ or $x = -1$

d) Graph the single function $y = \frac{13-4x}{x-5} - 2x - 1$ and identify the x-intercepts.





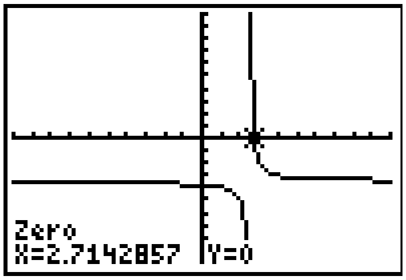
The equation has two solutions, $x = -2$ and $x = 4.5$.

$$
\begin{aligned}
2x + 1 &= \frac{13-4x}{x-5} \\
(2x+1)(x-5) &= 13 - 4x \\
2x^2 - 9x - 5 &= 13 - 4x \\
2x^2 - 5x - 18 &= 0 \\
(2x-9)(x+2) &= 0
\end{aligned}
$$

$x = 4.5$ or $x = -2$

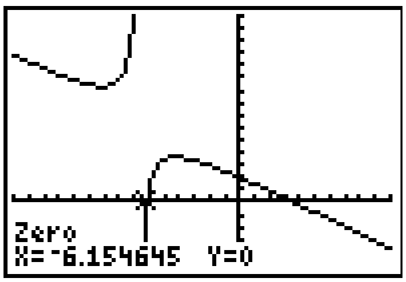
Chapter 9 Review Page 469 Question 10

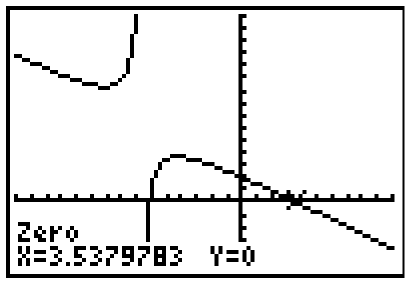
a) Graph the single function $y = \dfrac{x-4}{5-2x} - 3$ and identify the x-intercepts.



The equation has one solution, $x \approx 2.71$.

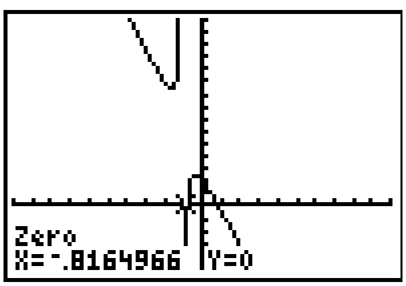
b) Graph the single function $y = \dfrac{x}{x+6.7} - 1.2x + 3.9$ and identify the x-intercepts.

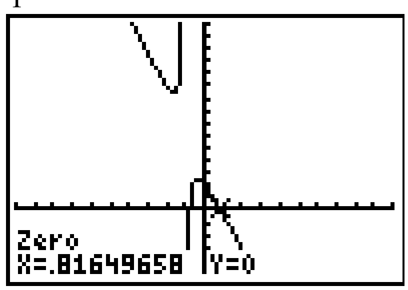




The equation has two solutions, $x \approx -6.15$ and $x \approx 3.54$.

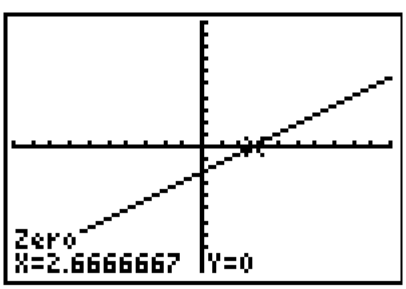
c) Graph the single function $y = \dfrac{5x+4}{x+1} - 3x - 2$ and identify the x-intercepts.





The equation has two solutions, $x \approx -0.82$ and $x \approx 0.82$.

d) Graph the single function $y = \dfrac{x^2-2x-8}{x+2} - \dfrac{1}{4}x + 2$ and identify the x-intercepts.

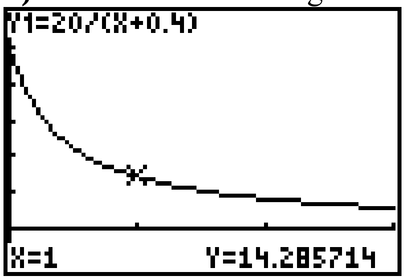


The equation has one solution, $x \approx 2.67$.

Chapter 9 Review Page 469 Question 11

a) The domain is $\{d \mid -0.4 < d \le 2.6, d \in \mathrm{R}\}$.

b) As the distance along the lever increases, less mass can be lifted.



c) The function has a vertical asymptote at its non-permissible value, $d = -0.4$. Since this is outside the domain, it has no effect on this situation.

The non-permissible value corresponds to the fulcrum point, which does not move when the lever is moved. As the mass gets closer to the fulcrum, it is possible to move a much heavier mass, but when the mass is on the fulcrum, it cannot be moved.

d) Substitute $m = 17.5$.

$$\begin{aligned} m &= \frac{20}{d+0.4} \\ 17.5 &= \frac{20}{d+0.4} \\ 17.5(d+0.4) &= 20 \\ 17.5d &= 13 \\ d &= 0.742... \end{aligned}$$

The lever can support a maximum possible mass of 17.5 kg at a distance of approximately 0.74 m from the fulcrum.

Chapter 9 Practice Test

Chapter 9 Practice Test Page 470 Question 1

A vertical asymptote at $x = 2$ corresponds to a factor of $x - 2$ in the denominator of the rational function: choice **C**.

Chapter 9 Practice Test Page 470 Question 2

The function $y = \frac{x^2 - 2x}{x^2 - 5x + 6}$ can be written as $y = \frac{x(x-2)}{(x-2)(x-3)}$. The graph will have a vertical asymptote at $x = 3$, a point of discontinuity at (2, –2), and an x-intercept of 0: choice **D**.

Chapter 9 Practice Test Page 470 Question 3

The function $y = -\frac{4}{x-2}$ has a vertical asymptote at $x = 2$. So, as x approaches 2, $|y|$ becomes very large: choice **C**.

Chapter 9 Practice Test Page 470 Question 4

The roots of $5 - x = \frac{x+2}{2x-3}$ can be found by determining the x-intercepts of the graph of $y = \frac{x+2}{2x-3} + x - 5$: choice **B**.

Chapter 9 Practice Test Page 470 Question 5

$$\begin{aligned} y &= \frac{6x-5}{x+7} \\ &= \frac{6x+42-42-5}{x+7} \\ &= \frac{6(x+7)-47}{x+7} \\ &= \frac{6(x+7)}{x+7} - \frac{47}{x+7} \\ &= 6 - \frac{47}{x+7} \\ &= -\frac{47}{x+7} + 6 \end{aligned}$$

Choice **D**.

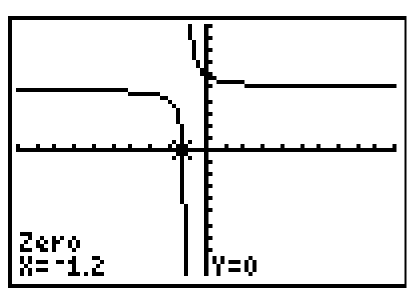
Chapter 9 Practice Test Page 470 Question 6

The graph of the function $y = \frac{x}{x^2 - x}$, or $y = \frac{x}{x(x-1)}$, will have a vertical asymptote at $x = 1$, a point of discontinuity at (0, –1), no intercepts. Choice **C**.

Chapter 9 Practice Test Page 470 Question 7

Graph the single function $y = \frac{x^2}{x+1} - x + 6$ and identify the x-intercepts.

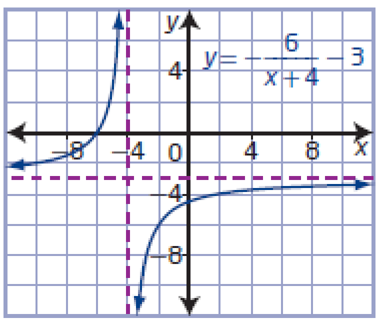
The equation has one solution, $x = -1.2$.



$$x-6=\frac{x^2}{x+1}$$
$$(x-6)(x+1)=x^2$$
$$x^2-5x-6=x^2$$
$$-5x=6$$
$$x=-\frac{6}{5} \quad \text{or} -1.2$$

Chapter 9 Practice Test Page 470 Question 8

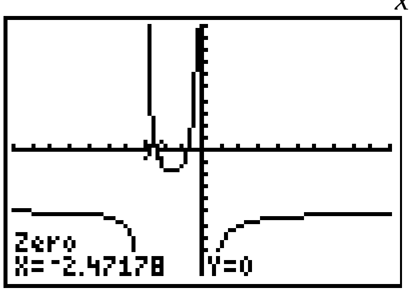
a) For $y=-\frac{6}{x+4}-3$, a = –6, h = –4, and k = –3. The graph of the base function $y=\frac{1}{x}$ is vertically stretched by a factor of 6, reflected in the x-axis, and translated 4 units to the left and 3 units down.

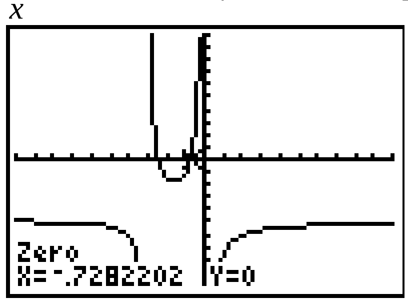


b) The domain is $\{x \mid x \neq -4, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq -3, y \in \mathrm{R}\}$. The vertical asymptote is at x = –4 and the horizontal asymptote is at y = –3.The x-intercept is –6. The y-intercept is –4.5.

Chapter 9 Practice Test Page 470 Question 9

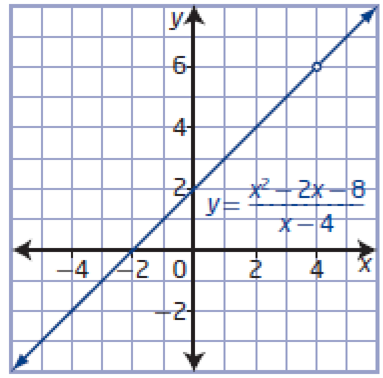
Graph the single function $y=\frac{2}{x+3}-\frac{3}{x}-5$ and identify the x-intercepts.





The equation has two solutions, $x \approx -2.47$ and $x \approx -0.73$.

Chapter 9 Practice Test Page 470 Question 10



a) Simplify the function.

$$\begin{aligned} y &= \frac{x^2 - 2x - 8}{x - 4} \\ &= \frac{(x-4)(x+2)}{x-4} \\ &= x + 2,\ x \neq 4 \end{aligned}$$

The graph has a point of discontinuity at (4, 6).

b) As x approaches 4, y approaches 6.

Chapter 9 Practice Test Page 471 Question 11

$$\begin{aligned} y &= \frac{2x^2 + 7x - 4}{x^2 + x - 12} \\ &= \frac{(2x-1)(x+4)}{(x+4)(x-3)} \\ &= \frac{2x-1}{x-3},\ x \neq -4, 3 \end{aligned}$$

The graph will have a vertical asymptote at $x = 3$. Substituting $x = -4$, the graph will have a point of discontinuity at $\left(-4, \frac{9}{7}\right)$. By substituting $y = 0$ and $x = 0$, the graph will have an x-intercept of $\frac{1}{2}$ and a y-intercept of $\frac{1}{3}$, respectively.

Chapter 9 Practice Test Page 471 Question 12

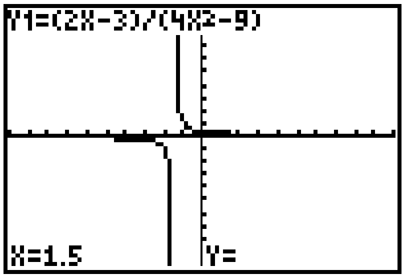
a) In factored form, $A(x) = \frac{x^2 - 9x}{x}$ is $A(x) = \frac{x(x-9)}{x}$. The graph has no vertical asymptotes, a point of discontinuity at (0, –9), and an x-intercept of 9: graph **D**.

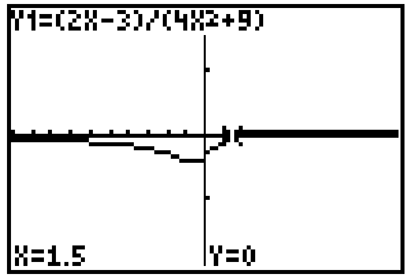
b) In factored form, $B(x) = \frac{x^2}{x^2 - 9}$ is $B(x) = \frac{x^2}{(x+3)(x-3)}$. The graph has a vertical asymptote at $x = \pm 3$, no points of discontinuity, and an x-intercept of 0: graph **A**.

c) In factored form, $C(x) = \frac{x^2 - 9}{x^2}$ is $C(x) = \frac{(x+3)(x-3)}{x^2}$. The graph has a vertical asymptote at $x = 0$, no points of discontinuity, and x-intercepts of –3 and 3: graph **B**.

d) In factored form, $D(x) = \dfrac{x^2}{x^2 - 9x}$ is $D(x) = \dfrac{x^2}{x(x-9)}$. The graph has a vertical asymptote at $x = 9$, a point of discontinuity at (0, 0), and no x-intercepts: graph **C**.

Chapter 9 Practice Test Page 471 Question 13





In factored form, $f(x) = \dfrac{2x-3}{4x^2-9}$ is $f(x) = \dfrac{2x-3}{(2x-3)(2x+3)}$. The function $g(x) = \dfrac{2x-3}{4x^2+9}$ cannot be factored.

The graph of $f(x)$ has a vertical asymptote at $x = -\dfrac{3}{2}$, since $2x + 3$ is a factor of only the denominator. The graph of $f(x)$ has a point of discontinuity at $\left(\dfrac{3}{2}, \dfrac{1}{6}\right)$. This is because $2x - 3$ is a factor of both the numerator and the denominator.

Since the denominator of $g(x)$ will never equal 0, the graph of $g(x)$ has no vertical asymptotes. Since there are no factors common to the numerator and denominator, the graph of $g(x)$ also has no points of discontinuity.

Chapter 9 Practice Test Page 471 Question 14

a) The equation has a non-permissible value of 2.

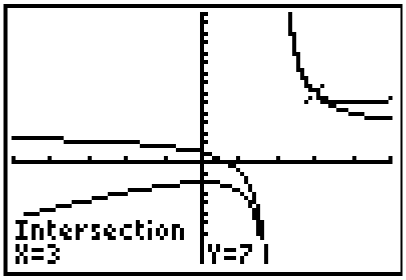
$$\frac{x^2}{x-2} - 2 = \frac{3x-2}{x-2}$$
$$x^2 - 2(x-2) = 3x - 2$$
$$x^2 - 2x + 4 = 3x - 2$$
$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$

$x = 2$ or $x = 3$

The solution is $x = 3$, since $x = 2$ is an extraneous root.

Alex should have excluded the extraneous root.

b) Graph the system of functions and find the points of intersection.

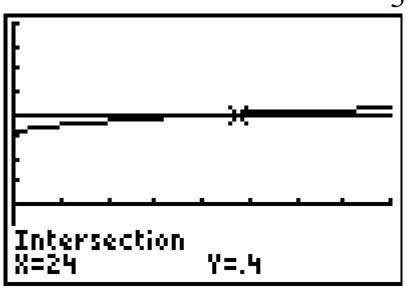


The solution is $x = 3$. The graphical method eliminates extraneous roots.

Chapter 9 Practice Test Page 471 Question 15

a) Given: 31 putts, 10 successes
If x represents the number of putts Jennifer takes from now on and she is successful on half of them, then a function that represents her average putting success rate is
$A = \dfrac{10 + 0.5x}{31 + x}$.

b) Graph the functions $A = \dfrac{10 + 0.5x}{31 + x}$ and $A = 0.40$ and find the point of intersection.

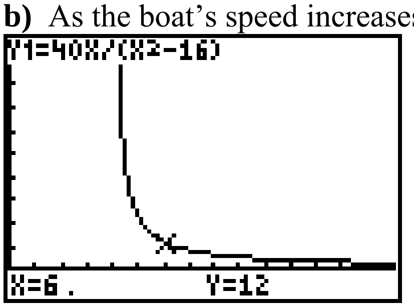


It will take 24 more putts for Jennifer to bring her average putting success rate up to her target.

Chapter 9 Practice Test Page 471 Question 16

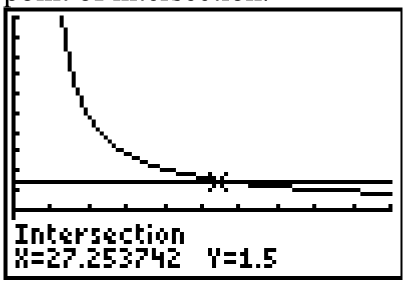
a) The domain is $\{v \mid v > 4, v \in \mathrm{R}\}$, since speed (and time) must be positive and $v = 4$ is a non-permissible value.

b) As the boat's speed increases, the total time for the round trip decreases.



c) As the boat's speed approaches 4 km/h, the time it takes for a round trip approaches infinity. The water flows at 4 km/h. If the boat's speed is less, the boat will never make the return trip, which is why there is an asymptote at $x = 4$.

d) The round trip will take 90 min, or 1.5 h. Graph $t = \dfrac{40v}{v^2 - 16}$ and $t = 1.5$ and find the point of intersection.



The boat speed needed is approximately 27.25 km/h.